

UNIVERSAL
LIBRARY



103 481

UNIVERSAL
LIBRARY

kansas city



public library

kansas city, missouri

Books will be issued only
on presentation of library card.

Please report lost cards and
change of residence promptly.

Card holders are responsible for
all books, records, films, pictures
or other library materials
checked out on their cards.

HOW TO

CALCULATE

QUICKLY

(the art of calculation)

BY HENRY STICKER

DOVER PUBLICATIONS, INC.

Copyright © 1945 by Essential Books.
Copyright © 1955 by Dover Publications, Inc.
All rights reserved under Pan American and
International Copyright Conventions.

Published in Canada by General Publishing Company, Ltd., 30 Lesmill Road, Don Mills, Toronto, Ontario.

Published in the United Kingdom by Constable and Company, Ltd., 10 Orange Street, London WC 2.

This Dover edition, first published in 1955, is an unabridged republication, with minor corrections, of the work originally published by Essential Books in 1945 under the title *The Art of Calculation*. It is reprinted through special arrangement with Duell, Sloan and Pearce, Inc.

Library of Congress Catalog Card Number: 56-3700

Manufactured in the United States of America
Dover Publications, Inc.
180 Varick Street
New York, N. Y. 10014

PREFACE

Arithmetic is a science, but calculation is an art. Science is knowledge—art is skill. You have all the knowledge you could possibly need to determine that 57 times 25 equals 1425, but if you are asked to multiply 57 by 25 and cannot do this mentally in just about one second, you are not adept at the art of calculation.

Genuine skill in the calculating art can be acquired by any person of ordinary intelligence, no matter what his schooling may have been. To develop such skill is the purpose of this book. Special forms of short, graded exercises, performed for the most part mentally, lead the student by easy steps to a point where he will possess really exceptional calculating ability.

For instance, if you will look at Exercise No. 371 on page ¹³³~~191~~, you will find that you are expected to perform mentally such multiplications as 696 times 858, 858 times 878, etc. These are not “trick” examples—the student who systematically performs the practice examples presented in this book will be able to do many kinds of examples of this degree of difficulty by his sheer ability to hold and manipulate figures *in his head*.

How is this skill developed? Essentially by developing *number sense*. Number sense consists in the ability to recognize the relations that exist between numbers considered as whole quantities, and to work with the thought of their broad relations always uppermost. Number sense is possessed by many people in all walks of life—particularly by accountants, bookkeepers, estimators, cashiers, storekeepers and the like. On the other hand, it is absent in many who have an excellent understanding of advanced mathe-

matics. The engineering professions are full of those who require slide rules to perform calculations which the average billing clerk would do mentally.

To give an example of what is meant by number sense, suppose you were asked to multiply mentally 11625 by 12. If you felt at all competent to try to do so, you would probably (unless you are the exceptional case) proceed like this: 12 times 5 is 60, remember 0 and carry 6; 12 times 2 is 24, put 0 before the other 0 and carry 3, etc. In this way you would eventually arrive at the correct answer—if you did not get all mixed up in the meantime; but at best you would take a long time, because number sense would have played no part whatever in your awkward method of approaching this very simple little problem.

Suppose now that we introduce a little of this number sense—suppose that instead of dealing with plain figures, you were told to imagine that you had sold twelve machines on each of which you made a commission of \$11.62½. As soon as money enters into the matter you immediately see the whole picture in a different light. If you were asked *approximately* how much your commissions amounted to, you would figure quick as a flash that 11 times 12 is 132, and you would probably answer instantly that you had made something over \$132. If you were then asked *how much* over \$132, you would either figure that 62½¢ are $\frac{5}{8}$ of one dollar, or else that this amount is equal to half a dollar plus $\frac{1}{8}$ of a dollar. You would not take long in determining that the excess over \$132 comes to \$7½, and that therefore the

total amount received would be \$139½ or \$139.50.

Why not apply to numbers "in the raw" the same methods that you use when dealing with small amounts of dollars and cents? It is no more difficult to multiply $11\frac{5}{8}$ thousands by 12 than $11\frac{5}{8}$ dollars. If $11\frac{5}{8}$ dollars times 12 is $139\frac{1}{2}$ dollars, then $11\frac{5}{8}$ thousands times 12 is $139\frac{1}{2}$ thousands, or 139,500.

From this illustration you may correctly infer that the person with number sense works very largely *from left to right* instead of from right to left. Left-to-right calculation is of the essence of number sense. Countless practical people know this, yet the art of left-to-right calculation is never taught in the schools, and is, in fact, rarely mentioned in books of any kind.

Step-by-step instruction and practice in this neglected art of left-to-right calculation constitutes the greater part of the substance of this book. Methods of this kind are applied not only to multiplication but to all the fundamental operations. By means of such methods, for instance, you learn to add two columns of figures at a time, and you even get a little practice in three-column addition. You are also taught comparable methods of subtraction and division.

In addition to the exercises having to do with left-to-right calculation, there are many that are based on an *extension of the multiplication table*. You are taught by easy stages to use all the numbers up to 25 as direct multipliers—that is to say, you acquire a *complete* knowledge of the multiplication table up to 25 times 25.

The subject of fractions is treated with special reference to the addition and subtraction of the

fractions that are most commonly met with in everyday work. The object here is to enable the student to memorize the answers to the kinds of problems that are ordinarily figured out over and over again.

The exercises dealing with decimals are designed to give the student a large workable fund of knowledge of the decimal equivalents of fractions. Memory work includes twelfths and sixteenths, and there is practice in the rapid calculation of thirty-seconds and twenty-fourths.

The final broad subject developed in this book is "short cuts." These are of the highest value in developing a general understanding of numbers.

The subject matter of this book is limited to the four fundamental operations, with the inclusion of fractions and decimals. No attempt is made to consider the various fields of arithmetical application. Skill in calculation pure and simple is the only goal.

The exercises, nearly four hundred in number, are for the most part very short. Few should take more than ten minutes to do, and many will take less. As progress is by graded steps, the instruction is in small "doses." The book, accordingly, can be used with profit whenever you happen to have a few free minutes. Its pocket size, moreover, makes it all the more suitable for odd-moment study.

Taken as a whole, this book will prove valuable to anybody engaged in work or study that requires any considerable amount of arithmetical calculation. It is especially recommended to heads of departments in industrial and commercial organizations, for general distribution to the members of their staffs.

CONTENTS

	PAGE
The Plan of This Book	2
ADDITION	
Addition in General	3
Adding Single Columns by Pairs, <i>starts on</i>	5
Adding Single Columns by Trios, <i>starts on</i>	22
Mental Addition of Large Numbers, <i>starts on</i>	42
Two-Column Addition, <i>starts on</i>	63
SUBTRACTION	
Subtraction in General	17
Left-to-Right Subtraction, <i>starts on</i>	19
MULTIPLICATION	
Multiplication in General	37
Factoring, <i>starts on</i>	55
Direct Multiplication by Numbers Greater than 12, <i>starts on</i>	56
Multiplying Three Figures by One, <i>starts on</i>	90
Multiplying Two Figures by Two, <i>starts on</i>	107
Multiplying Three Figures by Two, <i>starts on</i>	123
Multiplying Three Figures by Three, <i>starts on</i>	132
DIVISION	
Division in General	72
Direct Division by Numbers Greater than 12, <i>starts on</i>	79
Mental Division of Large Numbers, <i>starts on</i>	98
Division by Three Figures, <i>starts on</i>	104
Division by Two Figures, <i>starts on</i>	116
FRACTIONS	
Fractions in General	96
Addition and Subtraction of Fractions, <i>starts on</i>	97
DECIMALS	
Decimals in General	122
Decimal Equivalents of Fractions, <i>starts on</i>	123
SHORT CUTS	
Horizontal Addition	125
Combined Addition and Subtraction	127
Multiplying by a Near Number	130
Aliquot Parts in Multiplication	131
Simplifying the Multiplier	132
Multiplication by Factoring	134
Factors between 11 and 19	134
Multiplying by 11	135
Multiplying by 21, 31, 41, etc.	136
Squares of Numbers, <i>starts on</i>	137
Multiplying When Corresponding Orders Are Alike, <i>starts on</i>	139
Multiplying a Sum by a Difference	142
Multiplications Involving Fractions, <i>starts on</i>	142
Aliquot Parts in Division	143
Cubes of Numbers	144
Algebraic Multiplication	145
Table of Prime and Composite Numbers	146
ANSWERS	154

THE PLAN OF THIS BOOK

The subject matter here presented might have been divided into sections on addition, subtraction, multiplication, etc., in the manner usual to text-books on arithmetic. Because, however, of the special purpose of this book, no such division is made. The general plan is to have several branches proceed simultaneously. Progress is not from subject to subject but from less to more difficult calculation.

For each of the fundamental divisions of arithmetic there is a general introduction—for instance, *Addition in General* on page 3 . In these introductions the special objects sought are described, as well as the methods by which these objects are attained. The student, therefore, always has a clear view of the ultimate aims of his studies and knows how the work immediately in hand fits into the general plan.

Wherever anything new is introduced, it is clearly explained and illustrated. Usually the exercises that go with each explanation are spread through many succeeding pages. In a large number of cases the exercise calls for work with the numbers in a certain list or table (for instance, Table I on page 7). The same lists of numbers are used for various kinds of calculation. This method of presentation makes possible the remarkably great number (about 15,000) of practice examples that are included.

ADDITION IN GENERAL

Two main objects are sought. The first is to add by single columns, grouping three successive numbers at a time; the second is to add two columns at a time:

Take the following sum:

26

43

84

72

96

27

42

35

68

64

37

97

By the first method, starting at the top of the units' column, we would add these numbers thus: (sum of the first three figures) 13 (+ sum of the next three figures, 15) 28 (+ 15) 43 (+ 18) 61; write 1 and carry 6; (6 + 14) 20 (+ 18) 38 (+ 13) 51 (+ 18) 69; total, 691.

By the second method, starting at the top, we would add both columns simultaneously thus: (26 + 43) 69 (+ 84) 153 (+ 72) 225 (+ 96) 321 (+ 27) 348 (+ 42) 390 (+ 35) 425 (+ 68) 493 (+ 64) 557 (+ 37) 594 (+ 97) 691.

In actual practice, very rapid addition is possible by either method, and you will be left free

4 THE ART OF CALCULATION

to choose whichever you prefer. You should, however, learn both.

How do you proceed to learn these methods? You were taught—or should have been taught—at school that speed in addition is acquired by combining pairs of successive numbers that add up to 10. It is at this point that we start, because this is the simplest way in which grouped numbers can be added to a preceding sum. You are given short columns of numbers to be added by incidentally selecting such pairs of successive figures as make 10. In succeeding exercises the columns are lengthened, and you are also asked to group any pairs that add up to less than 10.

In the meantime, you will have been doing exercises in mentally adding all the numbers from 11 to 18 to all the numbers from 1 to 99. Since no pair of figures in a column can add to more than 18, this amount of practice will enable you to add *any* pair of successive figures in a column to a previous sum, and hence to add the entire column by taking two figures at a time.

You are similarly taught to add trios of numbers that make 10 or less than 10, and to add any number from 19 to 27 to any number from 1 to 99. With this practice you will be able to add *any* column by taking three figures at a time.

If you can quickly add any number from 1 to 27 to another number, you will not find it difficult to add numbers greater than 27 in the same manner. You are accordingly ready now to add two columns at a time. Exercises in this method are introduced, and these are gradually increased in difficulty.

Toward the end of the book there are some exercises in three-column addition—just enough to demonstrate that it will be possible for *you* to add this way if you wish to use this method.

There are examples in addition of still another kind. These are not included for practice in addition as such but have a special bearing on the art of multiplying mentally. We need not consider sums of this kind at this point.

You will note that in the exercises in one-column addition you are alternately instructed *to add from the top down* and *to add from the bottom up*. In practical work it is of course immaterial in which direction addition is performed. You should, however, be able to add with equal facility in either direction, and by alternating as suggested you will get the necessary practice.

Exercise No. 1

Pairs Adding to 10

Add the following columns by grouping pairs of numbers that make 10. *Add from the top down.*

Thus you would add the first column by saying to yourself: 7, 17, 22, 32.

Do not consciously repeat in your mind anything but the successive totals. That is to say, do *not* add this column thus: $7 + 10, 17, +5, 22, +10, 32$.

For another illustration of the correct method, take the second example. This is added thus: 8, 18, 20, 30.

Write your answers in succession on a piece of paper and compare them with the correct answers on page 154. (A good plan is to place the edge of your paper immediately under the examples, write the answers along this edge, and fold it under as it becomes used up.)

6 THE ART OF CALCULATION

1. 7	2. 8	3. 4	4. 5	5. 6	6. 5
6	9	5	2	4	5
4	1	5	8	6	3
5	2	5	4	3	6
1	3	4	1	2	4
9	7	6	9	8	8
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

7. 5	8. 3	9. 8	10. 6	11. 5	12. 9
4	2	2	9	5	6
6	7	9	1	3	4
6	3	8	5	2	8
3	1	1	4	4	1
7	2	9	6	6	7
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

13. 3	14. 1	15. 6	16. 6	17. 1	18. 7
7	9	4	3	3	6
6	9	4	7	7	2
2	1	5	2	9	8
8	5	4	2	3	5
8	4	3	5	7	5
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

19. 1	20. 1	21. 6	22. 3	23. 7	24. 4
9	5	4	4	5	9
4	5	7	6	5	1
3	9	6	4	3	3
9	4	3	6	6	2
1	6	7	3	2	8
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

Table I
Numbers from 1 to 99

1	8	15	22	29	36	43	50
57	64	71	78	85	92	99	6
13	20	27	34	41	48	55	62
69	76	83	90	97	4	11	18
25	32	39	46	53	60	67	74
81	88	95	2	9	16	23	30
37	44	51	58	65	72	79	86
93	7	14	21	28	35	42	49
56	63	70	77	84	91	98	5
12	19	26	33	40	47	54	61
68	75	82	89	96	3	10	17
24	31	38	45	52	59	66	73
80	87	94					

Exercise No. 2

Mental Addition

Add 11 to each of the numbers in Table I above.

Use *left-to-right* addition, which is performed by first adding the tens of one number to the whole of another. In other words, starting with the number in the table you first add 10 and then 1. A few illustrations will be in order:

15 + 11: say 15, 25, 26;

22 + 11: say 22, 32, 33;

29 + 11: say 29, 39, 40;

99 + 11: say 99, 109, 110.

Work down the columns—not across the page. Write down your answers and compare them with those on page 154.

Exercise No. 3

Pairs Adding to 10

Group all pairs of successive numbers that make 10.

Add from the bottom up.

1. 7	2. 6	3. 5	4. 9	5. 6	6. 3
8	4	2	7	7	1
4	5	5	6	9	6
6	2	4	4	1	4
5	4	6	8	3	4
3	5	6	8	4	1
5	4	7	9	6	8
5	1	3	1	3	2
1	2	4	1	8	9
8	8	8	7	5	6
2	7	2	5	2	4
<u>5</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>8</u>	<u>7</u>

7. 4	8. 8	9. 4	10. 6	11. 9	12. 3
7	2	4	5	8	7
3	9	3	7	8	6
8	1	2	3	2	6
3	5	4	4	7	1
2	3	6	2	1	2
2	8	1	8	9	7
8	5	6	9	6	6
1	5	4	1	5	4
9	2	9	3	5	5
1	6	3	2	5	5
<u>9</u>	<u>5</u>	<u>7</u>	<u>1</u>	<u>4</u>	<u>6</u>

13. 7	14. 3	15. 9	16. 1	17. 3	18. 6
4	7	1	8	6	9
6	8	6	7	4	1
3	2	3	5	2	7
2	8	7	5	8	7
6	5	5	6	5	3
4	5	4	7	1	2
1	8	6	3	4	1
8	2	4	5	1	5
3	7	3	4	9	2
7	1	2	4	3	9
<u>9</u>	<u>9</u>	<u>9</u>	<u>6</u>	<u>7</u>	<u>1</u>

Exercise No. 4

Mental Addition

Add 12 to the numbers in Table I on page 7.

To illustrate:

15 + 12: say 15, 25, 27;

22 + 12: say 22, 32, 34;

29 + 12: say 29, 39, 41;

99 + 12: say 99, 109, 111.

Exercise No. 5

Mental Addition

Add 13 to the numbers in Table I on page 7.

Exercise No. 6

Mental Addition

Add 14 to the numbers in Table I on page 7.

10 THE ART OF CALCULATION

Exercise No. 7

Mental Addition

Add 15 to the numbers in Table I on page 7.

Exercise No. 8

Pairs Adding to 10 or Less

The grouping of pairs of successive numbers is now to be extended to include any that add to less than 10 as well as any that add to 10. That is to say, as you add each column watch to see whether any two successive numbers add to either 10 or less than 10, and if they do, make one addition of them to the preceding sum.

For this exercise use the columns of numbers in Exercise No. 1 and compare your answers with those for Exercise No. 1. *Add from the top down.*

To illustrate, the first column is added: 7, 17, 23, 32; the second: 8, 18, 23, 30; the third: 9, 19, 29.

Exercise No. 9

Mental Addition

Add 16 to each of the numbers in Table I on page 7.

Exercise No. 10

Mental Addition

Add 17 to each of the numbers in Table I on page 7.

Exercise No. 11

Pairs Adding to 10 or Less

Add the columns in Exercise No. 3 by grouping all pairs of successive numbers that add to 10 or less than 10. *Add from the bottom up.*

Exercise No. 12

Mental Addition

Add 18 to each of the numbers in Table I on page 7.

Exercise No. 13

Adding Single Columns by Pairs

Add the following by single columns, taking pairs of successive numbers at a time. *Add from the top down.* The first example would be added: 5, 14, 25, write 5 and carry 2; 2, 12, 27, 36; answer 365.

1. 43	2. 29	3. 58	4. 87	5. 16
62	75	33	62	91
78	36	65	94	33
81	69	98	27	56
14	43	72	89	29
<u>87</u>	<u>16</u>	<u>45</u>	<u>74</u>	<u>32</u>

12 THE ART OF CALCULATION

6. 19	7. 48	8. 77	9. 36	10. 63
99	21	29	49	78
36	68	49	94	96
71	29	11	59	44
61	18	51	22	41
<u>41</u>	<u>25</u>	<u>53</u>	<u>27</u>	<u>88</u>

11. 33	12. 21	13. 34	14. 24	15. 16
39	79	43	14	44
43	74	27	11	49
51	85	53	15	54
55	63	17	75	49
<u>36</u>	<u>82</u>	<u>57</u>	<u>78</u>	<u>99</u>

16. 31	17. 28	18. 63	19. 32	20. 63
35	63	35	65	28
67	21	12	16	76
44	34	31	67	45
84	52	81	73	69
<u>42</u>	<u>56</u>	<u>15</u>	<u>55</u>	<u>62</u>

21. 85	22. 54	23. 14	24. 68	25. 69
56	42	27	42	28
75	68	54	28	45
37	13	85	34	37
73	99	59	83	71
<u>24</u>	<u>84</u>	<u>69</u>	<u>16</u>	<u>91</u>

Exercise No. 14

Mental Addition

Add 19 to each of the numbers in Table I on page 7.

Exercise No. 15

Adding Single Columns by Pairs

Add the following by single columns, taking pairs of successive numbers at a time. *Add from the bottom up.* The first example would be added: 11, 15, 27, 42, 49, 60, write 0 and carry 6; 6, 17, 24, 37, 43, 54, 62; answer, 620.

1. 27	2. 81	3. 92	4. 16	5. 29
64	28	92	14	27
32	75	29	14	25
85	43	86	31	25
46	96	54	97	32
29	57	18	65	19
78	51	68	29	76
64	89	62	79	51
31	75	11	73	12
43	42	86	22	84
75	54	53	58	33
<u>46</u>	<u>86</u>	<u>65</u>	<u>64</u>	<u>19</u>

14 THE ART OF CALCULATION

6. 43	7. 58	8. 74	9. 91	10. 99
51	54	69	85	13
38	62	65	91	96
36	49	74	76	13
37	47	71	85	87
33	36	58	82	96
41	34	47	69	93
87	52	35	58	87
62	98	63	37	69
23	73	31	74	47
95	34	84	42	75
<u>44</u>	<u>27</u>	<u>45</u>	<u>95</u>	<u>53</u>

11. 19	12. 39	13. 51	14. 63	15. 84
12	41	55	62	99
26	23	52	62	75
18	37	34	63	73
24	29	48	45	74
24	35	56	59	56
18	98	46	67	82
15	29	31	57	78
98	26	53	42	68
36	91	37	64	53
85	48	13	48	59
<u>49</u>	<u>96</u>	<u>59</u>	<u>24</u>	<u>57</u>

Exercise No. 16

Mental Addition

Add 20 to each of the numbers in Table I on page 7.

Exercise No. 17

Adding Single Columns by Pairs

Add the following by single columns, taking pairs of successive numbers at a time. *Add from the top down.*

1. 51	2. 42	3. 41	4. 34	5. 33
30	53	73	36	81
96	90	32	97	28
24	79	12	19	39
25	87	62	69	43
75	76	11	94	10
48	92	44	83	85
49	52	84	37	47
93	45	70	38	29
80	72	40	46	14
13	18	61	17	95
58	63	67	23	10
88	22	56	66	82
86	21	16	64	31
20	59	98	89	77
99	91	55	68	74
59	15	27	60	35
<u>65</u>	<u>78</u>	<u>54</u>	<u>23</u>	<u>84</u>

16 THE ART OF CALCULATION

6. 61	7. 34	8. 39	9. 36	10. 17
81	90	32	25	66
82	86	21	97	28
24	85	49	96	74
59	16	87	52	84
95	58	33	30	15
53	64	48	63	67
37	47	11	94	93
27	23	60	35	73
31	45	20	62	69
92	44	70	51	10
83	65	26	91	29
80	72	55	88	79
38	68	57	43	78
54	42	12	19	22
98	40	46	14	13
41	89	75	56	76
<u>77</u>	<u>99</u>	<u>18</u>	<u>42</u>	<u>39</u>

Exercise No. 18

Mental Addition

Add 21 to each of the numbers in Table I on page 7.

SUBTRACTION IN GENERAL

In keeping with the general object of this book, the succeeding exercises in subtraction are performed by left-to-right methods.

When subtraction is performed on paper there is no special advantage in working from left to right instead of from right to left. Paper practice in the former method, however, fits in with the broad purpose of developing number sense.

When it comes to doing subtraction mentally, the left-to-right method is natural and logical. Thus, if you had started the day with \$17.43 in your pocket, and if you wanted to figure without paper and pencil how much you had left after spending \$5.89, you would not be likely to start by subtracting 9 from 13. You would probably calculate that if you had spent the full \$6, you would have \$11.43 left, but that having spent 11¢ less than \$6, the remainder comes to 11¢ more than \$11.43, or \$11.54.

In considering the specific aims of these exercises in subtraction, look first at the written examples. If you will glance at the first exercise that follows, and which is included merely to familiarize you with the idea of working from left to right, you will see that in every case the figures in the subtrahend (lower number) are smaller than those in the minuend. The examples are all of the type of

and you can determine the answers faster than you can write them down. If, however, you take the example

$$\begin{array}{r} 685 \\ -356 \\ \hline \end{array}$$

and try to write the answer in the same way, you will run into trouble when you reach the final figures at the right because 6 is greater than 5. What to do about cases of this kind is the subject of the instruction. The exercises take into account the possible variations that may occur in numbers of two and three places.

The examples in mental subtraction are performed by methods altogether different from those that apply to written work. There are two such methods, of which one has already been illustrated. We subtracted \$5.89 from \$17.43 by taking \$6 from \$17.43 and then adding to \$11.43 the difference between \$6 and \$5.89, obtaining as our answer \$11.43 + \$.11, or \$11.54. To do the same example mentally by the other method, we calculate that if you had started with \$17 even, you would have \$11.11 left; but you had \$.43 more than \$17 at the start, and therefore the actual remainder is \$11.11 + \$.43, or \$11.54. One method is as good as the other. Examples are given that carry the practice in both methods as far as numbers involving hundreds of dollars and odd cents.

Incidentally, you should know that ordinary written subtraction is commonly performed by two entirely different methods—the *borrow*

method and the *carry* method. The borrow method is taught almost exclusively in this country today, but in times past the carry method had similar acceptance.

Take the example

$$\begin{array}{r} 856 \\ - 569 \\ \hline 287 \end{array}$$

To do this by the borrow method you reason: 9 from 16 leaves 7, 6 from 14 leaves 8, 5 from 7 leaves 2; answer, 287. To do the same example by the carry method you would say: 9 from 16 leaves 7, 7 from 15 leaves 8, 6 from 8 leaves 2; answer, 287.

You should understand both these methods (neither of which has any clear advantage over the other), though you continue to use regularly whichever one comes most naturally to you. In the illustrations given in this book the borrow method is followed because it is the more familiar to the majority of people.

Exercise No. 19

Left-to-Right Subtraction

Perform the following subtractions by directly writing your answers from left to right.

1. 67	2. 48	3. 41	4. 78	5. 64
<u>55</u>	<u>14</u>	<u>20</u>	<u>22</u>	<u>31</u>

6. 98	7. 53	8. 65	9. 28	10. 66
<u>20</u>	<u>41</u>	<u>52</u>	<u>16</u>	<u>45</u>

20 THE ART OF CALCULATION

11. 99 <u>92</u>	12. 69 <u>35</u>	13. 83 <u>31</u>	14. 32 <u>21</u>	15. 93 <u>41</u>
---------------------	---------------------	---------------------	---------------------	---------------------

Exercise No. 20

Left-to-Right Subtraction

Directly write your answers from left to right.

To take the first example, you simply note that 6 is greater than 4, and therefore the 5 in the minuend becomes a 4: 2 from 4 leaves 2 (writing 2), 6 from 14 leaves 8 (writing 8); answer 28.

1. 54 <u>26</u>	2. 47 <u>19</u>	3. 51 <u>39</u>	4. 46 <u>27</u>	5. 52 <u>37</u>
6. 84 <u>58</u>	7. 37 <u>18</u>	8. 35 <u>17</u>	9. 72 <u>24</u>	10. 50 <u>29</u>
11. 83 <u>44</u>	12. 56 <u>39</u>	13. 71 <u>45</u>	14. 96 <u>38</u>	15. 77 <u>49</u>
16. 94 <u>76</u>	17. 45 <u>16</u>	18. 48 <u>29</u>	19. 68 <u>39</u>	20. 71 <u>52</u>

Exercise No. 21

Mental Addition

Add 22 to each of the numbers in Table I on page 7.

Exercise No. 22

Trios that Add to 10 or Less

This exercise introduces the idea of taking in three suc-

SUBTRACTION IN GENERAL 21

cessive numbers at a glance. Every column contains four groups of three numbers each; each of these groups adds to 10 or less. Add by combining these groups. *Add from the top down.*

1. 27	2. 14	3. 64	4. 57	5. 34
21	11	21	31	31
11	12	13	12	11
45	33	44	56	54
41	21	42	21	42
13	13	22	23	13
65	25	43	56	52
12	21	32	12	31
12	24	33	12	22
25	35	78	45	44
11	12	11	21	31
<u>11</u>	<u>13</u>	<u>11</u>	<u>12</u>	<u>14</u>

6. 41	7. 62	8. 43	9. 21	10. 33
21	32	33	11	12
26	12	24	15	15
31	61	21	12	63
31	21	11	11	11
22	23	27	14	24
81	52	43	33	42
11	21	11	11	22
11	16	45	23	44
72	44	62	24	43
21	12	12	21	32
<u>13</u>	<u>14</u>	<u>15</u>	<u>25</u>	<u>33</u>

22 THE ART OF CALCULATION

Exercise No. 23

Left-to-Right Subtraction

Sight practice with pairs of three-place numbers. No borrowings are involved. Work from left to right.

1. 754	2. 827	3. 468	4. 659	5. 746
<u>233</u>	<u>614</u>	<u>235</u>	<u>338</u>	<u>415</u>

6. 928	7. 675	8. 558	9. 649	10. 458
<u>615</u>	<u>423</u>	<u>146</u>	<u>437</u>	<u>328</u>

11. 727	12. 898	13. 753	14. 462	15. 941
<u>605</u>	<u>457</u>	<u>321</u>	<u>111</u>	<u>720</u>

Exercise No. 24

Mental Addition

Add 23 to each of the numbers in Table I on page 7.

Exercise No. 25

Mental Addition

Add 24 to each of the numbers in Table I on page 7.

SUBTRACTION IN GENERAL 23

Exercise No. 26

Adding Single Columns by Pairs

Take successive pairs at a time. *Add from the top down.*

1. \$40.72	2. \$35.51	3. \$27.13	4. \$47.15
33.32	56.28	96.92	10.20
98.21	43.90	22.07	36.09
29.05	49.44	38.71	59.73
53.69	84.57	58.94	55.70
79.66	99.61	34.88	85.54
83.97	24.25	60.26	31.78
45.77	16.23	65.14	11.12
42.63	80.17	18.19	52.48
46.68	82.67	89.30	87.81
64.39	86.93	41.75	74.01
<u>37.62</u>	<u>91.76</u>	<u>50.95</u>	<u>25.60</u>

5. \$79.45	6. \$77.52	7. \$48.68	8. \$88.09
85.30	54.05	49.99	44.80
70.46	61.65	14.78	75.03
83.73	76.29	11.12	36.53
69.97	74.43	90.55	95.96
34.21	38.10	17.18	62.39
64.81	87.37	15.50	82.01
20.72	63.25	56.47	26.13
60.26	32.93	67.06	33.28
31.57	22.98	19.16	42.71
59.86	89.84	41.40	94.66
<u>58.35</u>	<u>91.23</u>	<u>56.15</u>	<u>10.34</u>

24 THE ART OF CALCULATION

Exercise No. 27

Left-to-Right Subtraction

In these examples, in the vertical pairs of figures at the extreme right the subtrahend is greater than the minuend, reducing by 1 the tens' figure of the minuend.

Taking the first example, we note that the tens' figure of the minuend will become a 4 instead of a 5; 5 from 7 leaves 2, 3 from 4 leaves 1, 9 from 14 leaves 5; answer 215.

1. $\begin{array}{r} 754 \\ 539 \\ \hline \end{array}$	2. $\begin{array}{r} 863 \\ 448 \\ \hline \end{array}$	3. $\begin{array}{r} 528 \\ 319 \\ \hline \end{array}$	4. $\begin{array}{r} 642 \\ 313 \\ \hline \end{array}$	5. $\begin{array}{r} 995 \\ 217 \\ \hline \end{array}$
--	--	--	--	--

6. $\begin{array}{r} 422 \\ 313 \\ \hline \end{array}$	7. $\begin{array}{r} 323 \\ 109 \\ \hline \end{array}$	8. $\begin{array}{r} 676 \\ 428 \\ \hline \end{array}$	9. $\begin{array}{r} 266 \\ 138 \\ \hline \end{array}$	10. $\begin{array}{r} 583 \\ 346 \\ \hline \end{array}$
--	--	--	--	---

11. $\begin{array}{r} 912 \\ 509 \\ \hline \end{array}$	12. $\begin{array}{r} 365 \\ 259 \\ \hline \end{array}$	13. $\begin{array}{r} 744 \\ 619 \\ \hline \end{array}$	14. $\begin{array}{r} 390 \\ 265 \\ \hline \end{array}$	15. $\begin{array}{r} 555 \\ 419 \\ \hline \end{array}$
---	---	---	---	---

16. $\begin{array}{r} 983 \\ 779 \\ \hline \end{array}$	17. $\begin{array}{r} 696 \\ 587 \\ \hline \end{array}$	18. $\begin{array}{r} 472 \\ 329 \\ \hline \end{array}$	19. $\begin{array}{r} 713 \\ 606 \\ \hline \end{array}$	20. $\begin{array}{r} 626 \\ 318 \\ \hline \end{array}$
---	---	---	---	---

21. $\begin{array}{r} 718 \\ 409 \\ \hline \end{array}$	22. $\begin{array}{r} 683 \\ 246 \\ \hline \end{array}$	23. $\begin{array}{r} 951 \\ 229 \\ \hline \end{array}$	24. $\begin{array}{r} 648 \\ 539 \\ \hline \end{array}$	25. $\begin{array}{r} 873 \\ 358 \\ \hline \end{array}$
---	---	---	---	---

26. $\begin{array}{r} 715 \\ 506 \\ \hline \end{array}$	27. $\begin{array}{r} 582 \\ 246 \\ \hline \end{array}$	28. $\begin{array}{r} 246 \\ 139 \\ \hline \end{array}$	29. $\begin{array}{r} 997 \\ 129 \\ \hline \end{array}$	30. $\begin{array}{r} 737 \\ 318 \\ \hline \end{array}$
---	---	---	---	---

Exercise No. 28

Mental Addition

Add 25 to each of the numbers in Table I on page 7.

Exercise No. 29**Mental Addition**

Add 26 to each of the numbers in Table I on page 7.

Exercise No. 30**Mental Addition**

Add 27 to each of the numbers in Table I on page 7.

Exercise No. 31**Trios that Add to 20 or Less**

In the separate columns of the following examples the successive groups of three figures add to some number between 11 and 20. Add by combining these groups of three. *Add from the top down.*

The first example would be added: 16, 30, 41, 61, write 1 and carry 6; 6, 18, 30, 46, 62; answer 621.

1. 23	2. 31	3. 12	4. 24	5. 24
46	46	84	64	74
67	46	89	74	78
21	12	33	35	35
55	24	43	45	55
58	97	78	95	78
22	13	13	14	14
54	73	37	45	44
95	86	99	75	99
12	23	13	25	25
69	57	88	65	35
<u>99</u>	<u>77</u>	<u>98</u>	<u>86</u>	<u>69</u>

6. 33	7. 32	8. 24	9. 34	10. 24
36	44	67	54	75
98	58	69	56	85
11	13	36	25	35
25	33	47	25	56
89	77	87	89	86
13	23	13	24	14
77	57	48	64	55
75	88	69	97	56
23	31	14	35	25
56	46	99	55	36
<u>69</u>	<u>68</u>	<u>98</u>	<u>67</u>	<u>77</u>

Exercise No. 32

Left-to-Right Subtraction

In the type of example given here we see by inspection that the subtrahend has a larger figure than the minuend in the tens' place, reducing by 1 the hundreds' figure of the minuend. To take the first example: 5 from 6 leaves 1, 9 from 15 leaves 6, 3 from 4 leaves 1; answer 161.

Subtract from left to right.

1. 754	2. 648	3. 262	4. 548	5. 629
<u>593</u>	<u>356</u>	<u>191</u>	<u>357</u>	<u>458</u>
6. 856	7. 435	8. 468	9. 914	10. 765
<u>792</u>	<u>183</u>	<u>271</u>	<u>291</u>	<u>481</u>

SUBTRACTION IN GENERAL 27

11. 787 <u>693</u>	12. 547 <u>160</u>	13. 341 <u>171</u>	14. 112 <u>51</u>	15. 783 <u>190</u>
-----------------------	-----------------------	-----------------------	----------------------	-----------------------

16. 486 <u>291</u>	17. 888 <u>494</u>	18. 489 <u>194</u>	19. 944 <u>452</u>	20. 842 <u>161</u>
-----------------------	-----------------------	-----------------------	-----------------------	-----------------------

Exercise No. 33

Left-to-Right Subtraction

In these examples the tens and the units are larger in the subtrahend than in the minuend, thus reducing by 1 both the hundreds and the tens of the minuend. Taking the first example: 2 from 6 leaves 4, 8 from 14 leaves 6, 9 from 14 leaves 5; answer, 465.

1. 754 <u>289</u>	2. 773 <u>194</u>	3. 413 <u>249</u>	4. 484 <u>298</u>	5. 342 <u>189</u>
----------------------	----------------------	----------------------	----------------------	----------------------

6. 626 <u>578</u>	7. 787 <u>298</u>	8. 383 <u>197</u>	9. 867 <u>379</u>	10. 672 <u>295</u>
----------------------	----------------------	----------------------	----------------------	-----------------------

11. 918 <u>589</u>	12. 666 <u>197</u>	13. 586 <u>298</u>	14. 232 <u>176</u>	15. 515 <u>299</u>
-----------------------	-----------------------	-----------------------	-----------------------	-----------------------

16. 353 <u>169</u>	17. 428 <u>179</u>	18. 856 <u>779</u>	19. 481 <u>192</u>	20. 318 <u>149</u>
-----------------------	-----------------------	-----------------------	-----------------------	-----------------------

Exercise No. 34

Adding Single Columns by Pairs

Add the following by single columns, taking pairs of successive numbers at a time. *Add from the bottom up.*

1. \$14.44	2. \$80.54	3. \$74.43	4. \$43.93
38.42	33.20	67.27	32.06
72.09	13.40	18.02	94.34
61.90	55.95	21.60	97.86
63.26	10.17	25.98	30.29
56.78	75.79	96.45	36.47
73.76	77.52	89.84	70.66
62.58	39.51	11.12	35.07
91.28	83.85	64.48	81.68
31.41	87.19	19.92	49.37
71.15	59.57	22.53	69.16
50.82	24.23	65.99	57.84
22.78	94.70	66.75	53.69
33.34	61.90	11.54	96.17
25.12	50.05	74.45	36.03
92.49	82.98	55.62	30.35
58.43	93.63	95.37	39.51
<u>75.64</u>	<u>20.67</u>	<u>72.71</u>	<u>48.15</u>

5. \$22.78	6. \$94.70	7. \$66.75	8. \$79.53
69.33	34.61	90.72	71.09
48.14	27.10	80.11	54.96
17.81	68.47	73.29	59.15
44.88	76.13	56.25	50.91
40.18	31.05	74.45	57.42
19.02	26.30	35.58	43.93
63.95	37.86	24.38	32.23
89.16	46.65	39.51	85.64
99.08	20.67	84.36	28.41
87.83	92.49	82.98	55.01
77.52	21.60	92.13	16.46
22.78	56.25	49.12	50.91
40.18	31.82	94.70	98.55
66.75	62.77	52.05	74.79
53.45	69.33	34.57	21.65
60.39	51.85	64.61	90.72
<u>71.09</u>	<u>48.15</u>	<u>27.10</u>	<u>80.06</u>

Exercise No. 35

Left-to-Right Subtraction

This exercise illustrates a principle: if a figure in the subtrahend is the same as the one above it in the minuend, the effect on the minuend will depend on whether or not a borrowing has been necessary with the next figure to the right.

In the first example we see that because 9 is greater than 4, the 5 in the minuend becomes a 4, and since 5 is greater than this the 7 in the minuend becomes a 6. We perform the subtraction thus: 3 from 6 leaves 3, 5 from 14 leaves 9, 9 from 14 leaves 5; answer, 395.

1. 754	2. 655	3. 251	4. 546	5. 592
<u>359</u>	<u>358</u>	<u>159</u>	<u>247</u>	<u>294</u>

30 THE ART OF CALCULATION

6. 862 <u>667</u>	7. 444 <u>146</u>	8. 968 <u>569</u>	9. 773 <u>279</u>	10. 763 <u>266</u>
11. 832 <u>536</u>	12. 233 <u>139</u>	13. 983 <u>488</u>	14. 572 <u>278</u>	15. 656 <u>357</u>
16. 395 <u>197</u>	17. 856 <u>659</u>	18. 645 <u>248</u>	19. 721 <u>428</u>	20. 941 <u>249</u>
21. 527 <u>329</u>	22. 863 <u>569</u>	23. 985 <u>389</u>	24. 267 <u>168</u>	25. 843 <u>448</u>

Exercise No. 36

Trios that Add to 27 or Less

The groups of three here add to numbers between 21 and 27. Add by combining these groups. *Add from the top down.*

1. 36	2. 63	3. 47	4. 65	5. 47
98	79	87	78	97
99	89	98	98	99
69	86	74	87	75
99	89	78	87	78
99	89	79	99	89
56	33	67	54	49
89	99	77	89	89
89	99	97	99	99
73	67	84	77	75
79	97	88	87	78
<u>99</u>	<u>97</u>	<u>99</u>	<u>88</u>	<u>78</u>

6. 55	7. 68	8. 56	9. 68	10. 56
88	88	87	88	98
89	88	99	99	98
77	85	78	96	78
78	99	88	98	89
98	99	89	98	99
65	57	96	68	66
89	98	97	89	78
89	99	98	99	89
87	76	78	96	84
98	87	78	97	88
<u>98</u>	<u>98</u>	<u>88</u>	<u>99</u>	<u>89</u>

Exercise No. 37

Left-to-Right Subtraction

In these examples another consideration arises: the tens' figure in the minuend is 0; when 1 is borrowed to make possible the subtraction of the units, the tens in the minuend become 9 and the hundreds are also reduced by 1.

To illustrate with the first example: 3 from 6 leaves 3, 5 from 9 leaves 4, 7 from 14 leaves 7; answer, 347.

Subtract from left to right.

1. 704	2. 307	3. 806	4. 204	5. 404
<u>357</u>	<u>118</u>	<u>457</u>	<u>126</u>	<u>297</u>
6. 808	7. 706	8. 308	9. 302	10. 203
<u>549</u>	<u>517</u>	<u>189</u>	<u>236</u>	<u>115</u>
11. 800	12. 501	13. 300	14. 805	15. 601
<u>585</u>	<u>323</u>	<u>122</u>	<u>796</u>	<u>374</u>

32 THE ART OF CALCULATION

16. 902	17. 500	18. 408	19. 700	20. 207
<u>793</u>	<u>386</u>	<u>159</u>	<u>466</u>	<u>178</u>

21. 807	22. 603	23. 200	24. 600	25. 300
<u>509</u>	<u>319</u>	<u>162</u>	<u>224</u>	<u>171</u>

Exercise No. 38

Adding Single Columns by Pairs

Take pairs of successive numbers at a time. *Add from the bottom up.*

1. \$5759.37	2. \$7856.21	3. \$6525.49
2186.62	2477.50	5214.44
4491.67	5843.84	8788.76
3848.60	3993.36	1115.81
6874.79	4751.85	2740.32
1831.04	9213.53	4569.82
1080.33	3363.26	9528.30
6461.73	9994.90	7271.70
<u>9823.34</u>	<u>9617.89</u>	<u>8983.55</u>

4. \$4142.97	5. \$6675.01	6. \$1916.46
4629.22	3508.07	2009.03
2089.83	5624.21	6538.82
9766.48	6039.10	8788.80
3367.72	7677.25	7531.01
9849.04	6393.03	8635.19
1623.26	6257.59	5096.58
4308.52	3646.51	1185.13
5354.34	9678.28	1714.55
4244.07	7170.27	4015.81
6874.79	3229.30	6422.37
<u>6118.91</u>	<u>4569.73</u>	<u>9947.94</u>

Exercise No. 39

Mental Subtraction

Use the method of making the subtrahend a round number. Subtract \$1 from the minuend and add to this the difference between \$1 and the given subtrahend.

Taking the first example: \$1 from \$5.18 leaves \$4.18; \$.83 from \$1 leaves \$.17; \$4.18 + \$.17 = \$4.35.

- | | |
|--------------------|--------------------|
| 1. \$5.18 - \$.83 | 11. \$3.22 - \$.93 |
| 2. \$6.42 - \$.83 | 12. \$7.37 - \$.61 |
| 3. \$1.89 - \$.95 | 13. \$4.56 - \$.97 |
| 4. \$2.47 - \$.99 | 14. \$6.87 - \$.91 |
| 5. \$7.48 - \$.56 | 15. \$2.21 - \$.65 |
| 6. \$8.29 - \$.66 | 16. \$4.86 - \$.97 |
| 7. \$3.18 - \$.87 | 17. \$3.32 - \$.64 |
| 8. \$7.27 - \$.43 | 18. \$7.75 - \$.83 |
| 9. \$4.19 - \$.49 | 19. \$4.12 - \$.63 |
| 10. \$3.53 - \$.77 | 20. \$6.23 - \$.26 |

Exercise No. 40

Adding Single Columns by Trios

Do the addition examples in Exercise No. 13 on page 11 by grouping three numbers at a time.

Taking the first example there presented, the following illustrates the method of adding: 13 (+12) 25, write 5 and carry 2; 2 (+17) 19, (+17) 36; answer, 365. Do not consciously repeat to yourself the individual amounts that you are adding, but only the successive total. *Add from the top down.*

34 THE ART OF CALCULATION

Exercise No. 41

Adding Single Columns by Pairs

1. \$7489.99	2. \$8356.24	3. \$2165.38
2897.66	4860.39	1034.96
7828.17	8084.05	8788.86
3519.16	2303.32	2922.64
2237.61	1891.45	4142.44
7170.27	4015.94	9062.57
5950.95	5843.08	9849.04
1209.63	9326.73	4768.79
8152.92	3646.51	1185.13
5354.14	5520.33	6772.76
7725.75	3104.60	1348.37
6101.98	4953.91	6039.62
5429.30	6772.76	1780.84
4414.57	5910.18	9134.96
7812.07	7170.06	8788.86
5056.24	9564.22	7755.63
2593.26	2075.27	4033.03
<u>4569.35</u>	<u>9236.74</u>	<u>8932.58</u>

4. \$8799.55	5. \$1319.16	6. \$8348.84
4437.14	5781.63	2538.82
9793.08	5266.88	2861.41
4223.59	3926.73	9809.50
3218.94	9156.24	5834.43
9564.65	2227.49	5340.33
6296.78	1207.54	5446.31
4569.35	7729.30	5115.71
7006.68	6772.11	8521.65
7976.92	9036.17	8074.89
3612.97	8909.50	2124.56
8765.77	2930.51	1507.23
5960.54	9964.75	2279.76
5546.31	7188.86	2858.34
4347.04	4147.61	8085.37
9570.06	1457.10	4884.44
6935.05	3218.94	8168.39
<u>6774.27</u>	<u>4913.26</u>	<u>7273.93</u>

Exercise No. 42**Mental Subtraction**

Perform the subtractions in Exercise No. 39 by using the method of making a round number of the minuend. That is, reduce the minuend to the next lower number of even dollars. Subtract the subtrahend from this and then add the excess of cents in the minuend.

Taking the first example (\$5.18 - \$.83): \$.83 from \$5 leaves \$4.17; \$4.17 + 18 = \$4.35.

36 THE ART OF CALCULATION

Exercise No. 43

Mental Subtraction

Perform the following subtractions mentally. Raise the subtrahend to the next larger number of even dollars.

- | | |
|---------------------|---------------------|
| 1. \$2.79 — \$1.86 | 11. \$5.53 — \$3.64 |
| 2. \$3.17 — \$1.97 | 12. \$2.62 — \$1.89 |
| 3. \$9.50 — \$6.69 | 13. \$3.05 — \$1.82 |
| 4. \$2.56 — \$1.91 | 14. \$8.28 — \$6.65 |
| 5. \$4.77 — \$2.81 | 15. \$8.10 — \$6.39 |
| 6. \$9.78 — \$3.94 | 16. \$5.15 — \$2.67 |
| 7. \$7.44 — \$4.49 | 17. \$4.47 — \$2.61 |
| 8. \$4.37 — \$2.72 | 18. \$7.93 — \$5.99 |
| 9. \$5.22 — \$2.98 | 19. \$5.40 — \$2.95 |
| 10. \$6.04 — \$5.33 | 20. \$3.23 — \$1.60 |

Exercise No. 44

Mental Subtraction

Do the examples in Exercise No. 43 by lowering the minuend to the next smaller number of even dollars.

MULTIPLICATION IN GENERAL

Multiplication is the heart's core of the art of calculation. In itself it constitutes an art about which a large volume might be written.

The multiplication exercises in this book have three main objects in view—first, to enable the student to use all numbers up to 25 as direct multipliers in written work; second, to teach him to multiply mentally any number up to 1000 by any other number up to 1000; third, to drill him in various short-cut methods that apply to particular cases.

The use of numbers up to 25 as direct multipliers may be illustrated by this example:

A	B
7648	7648
<u>1923</u>	<u>1923</u>
22944	175904
15296	<u>145312</u>
68832	14707104
<u>7648</u>	
14707104	

In Method A, which is here shown for comparison, the usual procedure is followed. In Method B the calculation is performed thus: $8 \times 23 = 184$, write 4 and carry 18; $4 \times 23 = 92$, $92 + 18 = 110$, write 0 and carry 11; $6 \times 23 = 138$, $138 + 11 = 149$, write 9 and carry 14; $7 \times 23 = 161$, $161 + 14 = 175$. Multiplication by 19 is done in the same way, and the partial products added.

38 THE ART OF CALCULATION

To multiply in the manner described it is of course necessary to acquire a knowledge of the multiplication table up to 25×25 . Instruction in this direction is given by very easy steps. There are several types of exercises leading to the same end.

Exercises in mental multiplication are similarly graded. You start by multiplying two figures by one, then two by two, then three by one, three by two, and finally three by three.

The subject of short cuts is highly specialized and need not detain us for the present.

Exercise No. 45

Mental Multiplication

Multiply by 2 the numbers in Table I on page 7. Proceed from left to right. A few examples of the method calculating will suffice.

$$32 \times 2: 30 \times 2 = 60, 2 \times 2 = 4, 60 + 4 = 64$$

$$45 \times 2: 40 \times 2 = 80, 5 \times 2 = 10, 80 + 10 = 90$$

$$49 \times 2: 40 \times 2 = 80, 9 \times 2 = 18, 80 + 18 = 98$$

$$99 \times 2: 90 \times 2 = 180, 9 \times 2 = 18, 180 + 18 = 198$$

Exercise No. 46

Mental Multiplication

Multiply mentally by 3 the numbers in Table I on page 7.

Exercise No. 47

Mental Multiplication

Multiply mentally by 4 the numbers in Table I on page 7.

MULTIPLICATION IN GENERAL 39

Exercise No. 48

Adding Single Columns by Pairs

Take pairs of successive numbers at a time. *Add from the bottom up.*

1. \$227976.55

491368.39

476170.02

804501.33

920950.63

512573.15

2. \$364631.71

291241.97

620314.57

378990.83

267278.30

586721.69

3. \$693505.74

822427.23

186620.98

871060.54

118577.94

996475.17

4. \$430413.93

525632.59

198886.28

651653.40

964295.81

480444.80

5. \$605465.38

599320.95

810064.74

112279.76

431275.17

890890.55

6. \$694235.68

483929.91

841653.40

344518.66

624133.37

364698.97

40 THE ART OF CALCULATION

Exercise No. 49

Mental Subtraction

Raise the subtrahend to the next larger number of even dollars.

- | | |
|--------------------|---------------------|
| 1. \$19.03 — \$.50 | 9. \$61.70 — \$.94 |
| 2. \$26.52 — \$.86 | 10. \$72.04 — \$.85 |
| 3. \$24.27 — \$.32 | 11. \$67.30 — \$.73 |
| 4. \$15.58 — \$.80 | 12. \$60.54 — \$.69 |
| 5. \$42.35 — \$.59 | 13. \$94.20 — \$.48 |
| 6. \$39.29 — \$.91 | 14. \$81.64 — \$.74 |
| 7. \$16.53 — \$.79 | 15. \$76.34 — \$.66 |
| 8. \$43.12 — \$.17 | 16. \$62.41 — \$.89 |

Exercise No. 50

Mental Multiplication

Multiply mentally by 5 the numbers in Table I on page 7.

Exercise No. 51

Mental Subtraction

Do the examples in Exercise No. 49 by reducing the minuend to the next smaller number of even dollars.

Exercise No. 52

Mental Multiplication

Multiply mentally by 6 the numbers in Table I on page 7.

Exercise No. 53

Mental Multiplication

Multiply mentally by 7 the numbers in Table I on page 7.

Exercise No. 54

Adding Single Columns by Pairs

Take pairs of successive numbers at a time. *Add from the top down.*

1. \$806054.65

681097.85

451866.93

431248.39

298291.24

322157.61

700177.25

714913.58

746789.23

569055.36

534011.98

281472.87

2. \$386942.35

933492.59

209507.09

751706.02

882750.78

305181.62

733115.33

379499.64

663265.52

444684.16

227976.86

377730.32

3. \$243130.39

158010.21

519794.95

893672.07

870485.02

834913.40

287919.76

697537.73

225942.35

435756.84

996168.05

164864.14

4. \$559663.93

882067.60

265254.65

332750.44

380353.71

462925.62

583492.78

411711.98

230882.09

911270.45

180190.66

744732.86

42 THE ART OF CALCULATION

Exercise No. 55

Mental Subtraction

Raise the subtrahend to the next larger number of even dollars.

- | | |
|---------------------|----------------------|
| 1. \$24.31 - \$4.55 | 9. \$96.15 - \$8.88 |
| 2. \$26.36 - \$7.50 | 10. \$87.04 - \$2.53 |
| 3. \$49.13 - \$4.62 | 11. \$79.19 - \$7.58 |
| 4. \$34.37 - \$7.98 | 12. \$59.42 - \$3.82 |
| 5. \$43.12 - \$1.70 | 13. \$99.05 - \$1.90 |
| 6. \$14.06 - \$7.86 | 14. \$77.24 - \$3.55 |
| 7. \$15.10 - \$2.88 | 15. \$67.60 - \$5.97 |
| 8. \$26.52 - \$6.89 | 16. \$72.07 - \$3.87 |

Exercise No. 56

Mental Multiplication

Multiply mentally by 8 the numbers in Table I on page 7.

Exercise No. 57

Adding Single Columns by Trios

Do the examples in Exercise No. 15 on page 12 by taking three successive numbers at a time. *Add from the top down.*

Exercise No. 58

Mental Subtraction

Do the examples in Exercise No. 55 by lowering the minuend to the next smaller number of even dollars.

Exercise No. 59

Addition of Partial Products

The type of exercise here presented has a bearing on mental multiplication. Thus the first example represents, in inverted position, the partial products we get when we multiply 15 by 53.

$$\begin{array}{r} 15 \\ 53 \\ \hline 45 \\ 750 \\ \hline 795 \end{array}$$

When partial products of this kind occur in mental multiplication you are of necessity compelled *to retain them in your mind*. Hence to develop your ability to do this kind of memory work, you are asked to read each example once and then write it three times on paper before you perform the mental addition.

Complete the mental addition before writing the answer. Work from left to right. Thus in doing the first example you would say to yourself: 750, 790, 795. In doing the second you would say: 620, 680, 682.

1. 750 <u>45</u>	2. 620 <u>62</u>	3. 470 <u>94</u>	4. 740 <u>74</u>	5. 520 <u>78</u>
6. 880 <u>44</u>	7. 720 <u>90</u>	8. 880 <u>66</u>	9. 960 <u>72</u>	10. 840 <u>72</u>
11. 850 <u>51</u>	12. 540 <u>81</u>	13. 570 <u>95</u>	14. 220 <u>88</u>	15. 910 <u>52</u>
16. 680 <u>34</u>	17. 980 <u>28</u>	18. 280 <u>84</u>	19. 640 <u>96</u>	20. 690 <u>92</u>
21. 760 <u>95</u>	22. 810 <u>54</u>	23. 750 <u>15</u>	24. 910 <u>78</u>	25. 580 <u>87</u>

Exercise No. 60

Mental Multiplication

Multiply mentally by 9 the numbers in Table I on page 7.

44 THE ART OF CALCULATION

Exercise No. 61

Mental Multiplication

Multiply mentally by 11 the numbers in Table I.

Exercise No. 62

Adding Single Columns by Pairs

Add from the bottom up.

1. \$698504.99	2. \$457012.91
845643.09	820823.58
761979.28	622529.46
401349.83	715303.47
740614.80	159363.96
553930.31	380272.36
896554.52	268195.94
975160.67	789234.17
417337.75	773286.20
882110.35	425922.98
116448.16	669001.18
477406.66	502733.07
801415.93	906396.55
340939.01	301831.05
380272.36	820889.23
656958.68	548620.61
882152.17	874185.10
<u>401304.99</u>	<u>761944.26</u>

MULTIPLICATION IN GENERAL 45

3. \$662533.75	4. \$473105.74
380277.80	141593.51
847236.82	111290.63
735356.57	897350.27
236569.58	379128.68
862061.88	966221.52
178735.81	644107.29
464385.34	104004.99
425919.44	266722.95
789249.94	987983.35
395497.48	183216.70
194426.67	295788.92
129066.25	336353.75
464347.56	578389.73
316085.34	740638.09
499498.27	236540.02
776980.14	159383.58
<u>518437.35</u>	<u>729128.36</u>

Exercise No. 63

Mental Subtraction

Raise the subtrahend to the next larger number of even dollars.

- | | |
|-----------------------|-----------------------|
| 1. \$83.37 - \$35.72 | 5. \$25.33 - \$10.65 |
| 2. \$68.20 - \$61.99 | 6. \$79.58 - \$51.84 |
| 3. \$97.48 - \$17.87 | 7. \$48.54 - \$20.61 |
| 4. \$64.41 - \$29.67 | 8. \$52.17 - \$30.32 |
| 9. \$91.28 - \$36.82 | 13. \$65.40 - \$14.93 |
| 10. \$76.42 - \$62.59 | 14. \$37.35 - \$28.82 |
| 11. \$55.30 - \$18.81 | 15. \$49.01 - \$21.85 |
| 12. \$95.12 - \$90.66 | 16. \$81.03 - \$41.16 |

Exercise No. 64**Continuous Addition Drill**

Count by 3's to 75.
 Count by 4's to 100.
 Count by 6's to 150.
 Count by 7's to 175.
 Count by 8's to 200.
 Count by 9's to 225.
 Count by 11's to 275.
 Count by 12's to 300.

Repeat this exercise three times.

Exercise No. 65**Mental Subtraction**

Do the examples in Exercise No. 63 by lowering the minuend to the next smaller number of even dollars.

Exercise No. 66**Mental Addition**

Read each of these examples once, write it three times and then add it mentally from left to right.

Be careful to think of the upper number in each case as something in the thousands and not as so many hundreds. Thus in the first example the upper number should be called one thousand seven hundred forty, *not* seventeen hundred forty. It is easier to think of comparatively small numbers as hundreds rather than as thousands plus hundreds, but this method of naming leads to trouble when dealing with larger numbers, and it is best to follow one uniform system.

1. 1740 <u>87</u>	2. 1650 <u>55</u>	3. 1080 <u>90</u>	4. 1280 <u>96</u>
5. 2430 <u>81</u>	6. 2560 <u>64</u>	7. 3690 <u>82</u>	8. 1120 <u>80</u>

MULTIPLICATION IN GENERAL 47

9. 1450 <u>87</u>	10. 1140 <u>95</u>	11. 1320 <u>88</u>	12. 1350 <u>78</u>
----------------------	-----------------------	-----------------------	-----------------------

13. 1340 <u>67</u>	14. 1320 <u>88</u>	15. 1920 <u>96</u>	16. 2340 <u>78</u>
-----------------------	-----------------------	-----------------------	-----------------------

17. 3680 <u>92</u>	18. 1080 <u>84</u>	19. 1950 <u>65</u>	20. 2520 <u>72</u>
-----------------------	-----------------------	-----------------------	-----------------------

Exercise No. 67

Mental Subtraction

Raise the subtrahend to the next larger number of even dollars.

- | | |
|----------------------|-----------------------|
| 1. \$855.30 - \$8.32 | 9. \$426.22 - \$7.78 |
| 2. \$844.16 - \$7.29 | 10. \$912.25 - \$5.33 |
| 3. \$671.46 - \$4.47 | 11. \$453.31 - \$5.60 |
| 4. \$834.06 - \$4.09 | 12. \$594.10 - \$7.23 |
| 5. \$642.02 - \$7.80 | 13. \$415.37 - \$7.91 |
| 6. \$836.11 - \$8.68 | 14. \$520.39 - \$9.76 |
| 7. \$862.21 - \$4.45 | 15. \$542.17 - \$8.55 |
| 8. \$532.13 - \$4.41 | 16. \$673.29 - \$9.44 |

Exercise No. 68

Adding Single Columns by Trios

Do the examples in Exercise No. 17 on page 15 by grouping three successive numbers at a time. *Add from the top down.*

Exercise No. 69

Mental Subtraction

Do the examples in Exercise No. 67 by reducing the minuend to the next smaller number of even dollars.

Table II
Numbers for Multiplication Table Drill

A	B	C	D	E	F	G	H	J	K	L	M
2	2	2	2	2	2	2	2	2	2	2	2
4	5	6	7	8	9	10	11	8	9	10	11
6	8	10	12	14	16	18	20	14	16	18	20
8	11	14	17	3	3	3	3	20	23	3	3
10	14	3	3	9	10	11	12	13	3	11	12
12	3	7	8	15	17	19	21	9	10	19	21
14	6	11	13	4	4	4	4	15	17	4	4
3	9	15	4	10	11	12	13	21	4	12	13
5	12	4	9	16	18	20	5	4	11	20	22
7	15	8	14	5	5	5	14	10	18	5	5
9	4	12	5	11	12	13	6	16	5	13	14
11	7	16	10	17	19	6	15	22	12	21	23
13	10	5	15	6	6	14	7	5	19	6	6
	13	9	6	12	13	7	16	11	6	14	15
		13	11	18	7	15	8	17	13	22	24
			16	7	14	8	17	6	20	7	7
				13	8	16	9	12	7	15	16
					15	9	18	18	14	23	25
						17	10	7	21	8	8
							19	13	8	16	17
								19	15	24	9
									22	9	18
										17	10
											19

Exercise No. 70

Multiplication Table Drill

Use Table II on this page. Multiply the numbers in Column A successively by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. Repeat this exercise three times.

Exercise No. 71

Mental Subtraction

Raise the subtrahend to the next larger number of even dollars, and raise this amount in turn to an even \$100. Thus, taking the first example: \$100 from \$365.42 leaves \$265.42; \$265.42 + \$11 (difference between \$100 and \$89) equals \$276.42; \$276.42 + \$.27 = \$276.69.

- | | |
|-----------------------|------------------------|
| 1. \$365.42 - \$88.73 | 9. \$459.48 - \$87.55 |
| 2. \$950.49 - \$94.98 | 10. \$553.18 - \$81.64 |
| 3. \$723.67 - \$40.77 | 11. \$416.07 - \$29.19 |
| 4. \$614.15 - \$93.79 | 12. \$426.22 - \$95.78 |
| 5. \$858.51 - \$84.72 | 13. \$912.25 - \$33.63 |
| 6. \$928.36 - \$36.82 | 14. \$753.46 - \$56.57 |
| 7. \$413.54 - \$86.61 | 15. \$831.05 - \$60.85 |
| 8. \$342.21 - \$96.62 | 16. \$743.16 - \$68.29 |

Exercise No. 72

Adding Single Columns by Trios

Do the examples in Exercise No. 22 on page 20 by grouping three successive numbers at a time. *Add from the bottom up.*

Table III

Numbers to Be Multiplied

- | | | |
|-----------|------------|------------|
| 1. 111315 | 6. 171922 | 11. 222572 |
| 2. 111417 | 7. 182123 | 12. 541418 |
| 3. 121416 | 8. 897254 | 13. 192389 |
| 4. 121518 | 9. 248963 | 14. 151924 |
| 5. 541316 | 10. 258163 | 15. 212481 |

50 THE ART OF CALCULATION

Exercise No. 73

Written Multiplication

Multiply the numbers in Table III by 6789.

Exercise No. 74

Mental Addition

Read each of the following examples once, write it three times and then add it mentally from left to right.

Think of the upper number in each case as being in the thousands and not the hundreds.

The first example would be added: 1280, 1480, 1536. In other words, take the first number as a whole, and then add to it successively the hundreds, tens and units of the second number.

1. 1280	2. 4410	3. 1960	4. 1380
<u>256</u>	<u>196</u>	<u>686</u>	<u>115</u>

5. 4620	6. 3060	7. 6510	8. 4150
<u>693</u>	<u>170</u>	<u>837</u>	<u>664</u>

9. 4080	10. 1110	11. 6480	12. 1450
<u>204</u>	<u>185</u>	<u>144</u>	<u>174</u>

13. 1640	14. 3350	15. 5150	16. 3510
<u>246</u>	<u>268</u>	<u>344</u>	<u>351</u>

17. 3040	18. 8080	19. 1240	20. 2250
<u>304</u>	<u>528</u>	<u>372</u>	<u>405</u>

MULTIPLICATION IN GENERAL 51

Exercise No. 75

Mental Subtraction

Do the examples in Exercise No. 71 on page 49 by lowering the minuend. Reduce it to the next smaller number of even dollars. Taking the first example: $\$300 - \88.73 leaves $\$211.27$; $\$211.27 + \$65 = \$276.27$; $\$276.27 + \$42 = \$276.69$.

Exercise No. 76

Adding Single Columns by Trios

Do the examples in Exercise No. 26 on page 23 by grouping three successive numbers at a time. *Add from the top down.*

Exercise No. 77

Mental Multiplication

Multiply mentally by 12 the numbers in Table I on page 7.

Exercise No. 78

Adding Single Columns by Trios

Do the examples in Exercise No. 34 on page 28 by grouping three successive numbers at a time.

Exercise No. 79

Mental Subtraction

Raise the subtrahend to the next larger number of even hundreds of dollars.

1. $\$950.49 - \498.65

2. $\$646.43 - \456.57

3. $\$520.39 - \176.42

4. $\$821.13 - \468.54

5. $\$769.14 - \580.93

6. $\$831.05 - \685.34

7. $\$821.45 - \529.48

8. $\$862.39 - \197.76

- | | |
|-------------------------|-------------------------|
| 9. \$318.32 - \$181.64 | 13. \$416.07 - \$219.44 |
| 10. \$636.09 - \$549.95 | 14. \$640.02 - \$493.79 |
| 11. \$714.10 - \$273.65 | 15. \$746.14 - \$159.93 |
| 12. \$821.45 - \$599.97 | 16. \$752.30 - \$183.81 |

Exercise No. 80

Mental Addition

Read each of the following examples once, write it three times and then add it mentally from left to right. The first example would be added: 16530, 17030, 17081.

- | | | |
|-------------------------|-------------------------|-------------------------|
| 1. 16530
<u>551</u> | 2. 12930
<u>431</u> | 3. 24920
<u>623</u> |
| 4. 22080
<u>552</u> | 5. 37150
<u>743</u> | 6. 33650
<u>673</u> |
| 7. 51780
<u>863</u> | 8. 44460
<u>741</u> | 9. 67340
<u>962</u> |
| 10. 61810
<u>883</u> | 11. 19360
<u>242</u> | 12. 12160
<u>152</u> |
| 13. 76960
<u>962</u> | 14. 32670
<u>363</u> | 15. 25380
<u>282</u> |
| 16. 12690
<u>141</u> | 17. 15320
<u>766</u> | 18. 19620
<u>654</u> |
| 19. 21720
<u>543</u> | 20. 46650
<u>933</u> | 21. 44160
<u>736</u> |

Exercise No. 81**Written Multiplication**

Multiply by 1112 each of the numbers in Table III on page 49. Wherever there occurs in the multiplicand a pair of figures that may be considered as 11 or 12, make one multiplication of this instead of two, and accordingly write down two figures in the partial product. Taking the first example:

$$\begin{array}{r}
 111315 \\
 \underline{1112} \\
 1335780 \\
 1224465 \\
 \hline
 123782280
 \end{array}$$

111315 is successively multiplied (from right to left) by 12 and 11 thus: $5 \times 12 = 60$, write 0 and carry 6; $1 \times 12 = 12$, $12 + 6 = 18$, write 8 and carry 1; $3 \times 12 = 36$, $36 + 1 = 37$, write 7 and carry 3; $11 \times 12 = 132$, $132 + 3 = 135$, write 35 and carry 1; $1 \times 12 = 12$, $12 + 1 = 13$, write 13. Multiplication by 11 is carried out in the same way.

In doing these examples be watchful about placing the second partial product *two* places to the left of the first.

Exercise No. 82**Adding Single Columns by Trios**

Do the examples in Exercise No. 38 on page 32 by grouping three successive numbers at a time. *Add from the bottom up.*

Exercise No. 83**Mental Subtraction**

Do the examples in Exercise No. 79 on page 51 by lowering the minuend to the next smaller number of even hundreds of dollars.

54 THE ART OF CALCULATION

Exercise No. 84

Mental Addition

Read each of the following examples once, write it three times and then add it mentally from left to right.

Add in turn the thousands, hundreds, tens and units to the upper number. In doing the first example you should say to yourself something like the following: $18360 + 1224$, 19360 ; $19360 + 224$, 19560 ; $19560 + 24$, 19584 .

$$\begin{array}{r} 1. \ 18360 \\ \quad 1224 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \ 21630 \\ \quad 2163 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \ 24960 \\ \quad 3328 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \ 18820 \\ \quad 5646 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \ 16260 \\ \quad 1084 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \ 19530 \\ \quad 1953 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \ 21360 \\ \quad 2848 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \ 16420 \\ \quad 4926 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \ 18640 \\ \quad 6524 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \ 10290 \\ \quad 2401 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \ 13530 \\ \quad 3608 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \ 16860 \\ \quad 5058 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \ 29240 \\ \quad 1462 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \ 33680 \\ \quad 2526 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \ 28590 \\ \quad 4765 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \ 13230 \\ \quad 3969 \\ \hline \end{array}$$

$$\begin{array}{r} 17. \ 26520 \\ \quad 1326 \\ \hline \end{array}$$

$$\begin{array}{r} 18. \ 28840 \\ \quad 2163 \\ \hline \end{array}$$

$$\begin{array}{r} 19. \ 24960 \\ \quad 4160 \\ \hline \end{array}$$

$$\begin{array}{r} 20. \ 28290 \\ \quad 5658 \\ \hline \end{array}$$

$$\begin{array}{r} 21. \ 14120 \\ \quad 2118 \\ \hline \end{array}$$

Exercise No. 85

Continuous Addition Drill

Count by 4's to 100.

Count by 6's to 150.

Count by 7's to 175.

Count by 8's to 200.

Count by 9's to 225.

Count by 11's to 275.

Count by 12's to 300.

Count by 13's to 325.

Repeat this exercise three times.

Exercise No. 86

Adding Single Columns by Trios

Do the examples in Exercise No. 41 on page 34 by grouping three successive numbers at a time. *Add from the top down.*

Exercise No. 87

Factoring

When numbers are multiplied together, they are considered *factors* of the resulting *product*. Thus 2 and 3 are factors of 6, and 3 and 5 are factors of 15.

Factoring a number is the process of resolving the number into the factors that will produce the number when multiplied together. Thus 36 may be factored as 2×18 , or as 3×12 , or as 4×9 , or as 6×6 .*

Any number that can be resolved into factors is called a *composite* number.

A *prime* number is one that has no factors besides itself and 1. Thus, 1, 2, 3, 5, 7, 11, 13, etc. are prime numbers.

* If it were required to give the *prime* factors of 36, these would be $2 \times 2 \times 3 \times 3$, but factoring into prime numbers has nothing to do with the purposes of this book.

On the pages starting with 146 will be found a table which analyzes all prime and composite numbers up to 625. You will be taught gradually to familiarize yourself with this entire table. The purpose of this is to help you to recognize quickly the character of these numbers—to enable you to multiply rapidly the factors that produce any of them, or to separate any of them into such factors.

Of special importance in this table are the numbers printed in italic type, since these can be produced by two factors each of which is 25 or less.

It is quite commonly appreciated that very small numbers have a definite individuality which grows out of the many associations built up around them in our minds. The individual character of higher numbers becomes similarly apparent and unforgettable when we single them out for particular attention.

For the first exercise in factoring read the first two columns of the table on page 146, and then write these from memory (or calculation) in the same form.

In studying the table note that each composite number is factored by first taking the smaller factors in the order of their size, and that the combinations are not repeated. Thus the separate ways of factoring 48 are given as 2×24 , 3×16 , 4×12 and 6×8 . These combinations are not repeated as 8×6 , 12×4 , 16×3 , and 24×2 .

Exercise No. 88

Multiplication Table Drill

Use Table II on page 48.

Multiply the numbers in Column A successively by 3, 4, 6, 7, 8, 9, 11, 12 and 13.

Repeat this exercise three times.

This exercise takes us the first step beyond the custom-

ary limits of the multiplication table, which ordinarily goes no farther than 12×12 . Succeeding examples will enable you to memorize the products of all pairs of numbers up to 25×25 .

No multiplication table, as such, is presented in this book, because learning the products of higher factors by sheer power of memory is extremely difficult. On the other hand, when you are put over and over again to the necessity of figuring out these higher combinations for yourself, they soon come to stick firmly in the mind.

Exercise No. 89

Mental Addition

Read each of the following examples once, write it three times, and then add it mentally from left to right. The first example would be added: 165300, 170300, 170810.

$$\begin{array}{r} 1. \ 165300 \\ \quad 5510 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \ 129300 \\ \quad 4310 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \ 249200 \\ \quad 6230 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \ 220800 \\ \quad 5520 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \ 371500 \\ \quad 7430 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \ 336500 \\ \quad 6730 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \ 517800 \\ \quad 8630 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \ 444600 \\ \quad 7410 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \ 673400 \\ \quad 9620 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \ 618100 \\ \quad 8830 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \ 193600 \\ \quad 2420 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \ 121600 \\ \quad 1520 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \ 769600 \\ \quad 9620 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \ 326700 \\ \quad 3630 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \ 253800 \\ \quad 2820 \\ \hline \end{array}$$

58 THE ART OF CALCULATION

$$\begin{array}{r} 16. \ 126900 \\ \underline{\quad 1410} \end{array}$$

$$\begin{array}{r} 17. \ 153200 \\ \underline{\quad 7660} \end{array}$$

$$\begin{array}{r} 18. \ 196200 \\ \underline{\quad 6540} \end{array}$$

$$\begin{array}{r} 19. \ 217200 \\ \underline{\quad 5430} \end{array}$$

$$\begin{array}{r} 20. \ 456500 \\ \underline{\quad 9330} \end{array}$$

$$\begin{array}{r} 21. \ 441600 \\ \underline{\quad 7360} \end{array}$$

Exercise No. 90

Mental Multiplication

Multiply mentally by 13 the numbers in Table I on page 7.

In working with numbers from 80 upward, immediately name 1000 as the first part of the product. Thus 83×13 is 1040, (+39) 1079; 97×13 is 1170, 1261.

Exercise No. 91

Adding Single Columns by Trios

Do the examples in Exercise No. 48 on page 39 by grouping three successive numbers at a time. *Add from the bottom up.*

Exercise No. 92

Factoring

Read the table on page 146 from 31 to 72 inclusive, and then write it in the same form.

Exercise No. 93

Mental Addition

Read each of the following examples once, write it three times and then add it mentally from left to right.

Add in turn the tens of thousands, thousands, hundreds and tens to the upper number. The first example would be added: 183600, 193600, 195600, 195840.

MULTIPLICATION IN GENERAL 59

1. 183600 <u>12240</u>	2. 216300 <u>21630</u>	3. 249600 <u>33280</u>
4. 188200 <u>56460</u>	5. 162600 <u>10840</u>	6. 195300 <u>19530</u>
7. 213600 <u>28480</u>	8. 164200 <u>49260</u>	9. 186400 <u>65240</u>
10. 102900 <u>24010</u>	11. 135300 <u>36080</u>	12. 168600 <u>50580</u>
13. 292400 <u>14620</u>	14. 336800 <u>25260</u>	15. 285900 <u>47650</u>
16. 132300 <u>39690</u>	17. 265200 <u>13260</u>	18. 288400 <u>21630</u>
19. 249600 <u>41600</u>	20. 282900 <u>56580</u>	21. 141200 <u>21180</u>

Exercise No. 94

Written Multiplication

Multiply by 1213 each of the numbers in Table III on page 49. Wherever there occurs in the multiplicand a pair of figures that may be considered as 11, 12 or 13, make one multiplication of this instead of two, and write two figures in the partial product. Thus, taking the first example, we successively multiply 15, 13 and 11 by 13 and again by 12. The partial products are accordingly written in two lines instead of the customary four.

60 THE ART OF CALCULATION

Exercise No. 95

Adding Single Columns by Trios

Do the examples in Exercise No. 54 on page 41 by grouping three successive numbers at a time. *Add from the top down.*

Exercise No. 96

Factoring

Factor the numbers from 54 to 92 inclusive in the form shown in the table on page 146.

Exercise No. 97

Mental Addition

Read each of the following examples once, write it three times and then add it mentally from left to right.

Add the whole of the second number to the first before considering the third. Repeat to yourself several times the sum of the first and second if you find this necessary.

The third example would be added: 36300, 39300, 39930; (repeat 39930, 39930); 39930, 40030, 40051.

$$\begin{array}{r} 1. \ 10100 \\ \quad 1010 \\ \quad \quad 101 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \ 22200 \\ \quad 2220 \\ \quad \quad 222 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \ 36300 \\ \quad 3630 \\ \quad \quad 121 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \ 52400 \\ \quad 5240 \\ \quad \quad 262 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \ 70500 \\ \quad 7050 \\ \quad \quad 141 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \ 90600 \\ \quad 1510 \\ \quad \quad 302 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \ 19100 \\ \quad 9950 \\ \quad \quad 382 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \ 20200 \\ \quad 1010 \\ \quad \quad 101 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \ 33300 \\ \quad 2220 \\ \quad \quad 222 \\ \hline \end{array}$$

10. 48400 3630 <u>121</u>	11. 65500 5240 <u>262</u>	12. 84600 7050 <u>141</u>
13. 18100 7240 <u>181</u>	14. 38200 9050 <u>905</u>	15. 20200 4040 <u>202</u>
16. 42400 6360 <u>424</u>	17. 66600 8880 <u>666</u>	18. 40400 4040 <u>404</u>
19. 33600 3360 <u>336</u>	20. 88800 8880 <u>222</u>	21. 30300 9090 <u>303</u>

Exercise No. 98

Continuous Addition Drill

Count by 6's to 150.
 Count by 7's to 175.
 Count by 8's to 200.
 Count by 9's to 225.
 Count by 11's to 275.
 Count by 12's to 300.
 Count by 13's to 325.
 Count by 14's to 350.

Repeat this exercise three times.

Exercise No. 99

Adding Single Columns by Trios

Do the examples in Exercise No. 62 on page 44 by grouping three successive numbers at a time. *Add from the bottom up.*

62 THE ART OF CALCULATION

Exercise No. 100

Factoring

Factor the numbers from 73 to 111 inclusive in the form shown in the table on page 146.

Exercise No. 101

Mental Addition

Read each of the following examples once, write it three times and then add it mentally from left to right.

The first example would be added: 26200, 33200, 34000, 34060; 34060, 36060, 36156.

$$\begin{array}{r} 1. \ 26200 \\ \quad 7860 \\ \quad \underline{2096} \end{array}$$

$$\begin{array}{r} 2. \ 48400 \\ \quad 9680 \\ \quad \underline{1210} \end{array}$$

$$\begin{array}{r} 3. \ 69900 \\ \quad 9320 \\ \quad \underline{1398} \end{array}$$

$$\begin{array}{r} 4. \ 12100 \\ \quad 9680 \\ \quad \underline{1089} \end{array}$$

$$\begin{array}{r} 5. \ 26400 \\ \quad 9240 \\ \quad \underline{1056} \end{array}$$

$$\begin{array}{r} 6. \ 42900 \\ \quad 8580 \\ \quad \underline{1144} \end{array}$$

$$\begin{array}{r} 7. \ 61600 \\ \quad 9240 \\ \quad \underline{1078} \end{array}$$

$$\begin{array}{r} 8. \ 82500 \\ \quad 9900 \\ \quad \underline{1155} \end{array}$$

$$\begin{array}{r} 9. \ 88000 \\ \quad 8800 \\ \quad \underline{1056} \end{array}$$

$$\begin{array}{r} 10. \ 93500 \\ \quad 9350 \\ \quad \underline{1122} \end{array}$$

$$\begin{array}{r} 11. \ 98000 \\ \quad 9800 \\ \quad \underline{1188} \end{array}$$

$$\begin{array}{r} 12. \ 73200 \\ \quad 9760 \\ \quad \underline{1098} \end{array}$$

$$\begin{array}{r} 13. \ 93100 \\ \quad 9310 \\ \quad \underline{1064} \end{array}$$

$$\begin{array}{r} 14. \ 97600 \\ \quad 9760 \\ \quad \underline{1220} \end{array}$$

$$\begin{array}{r} 15. \ 71000 \\ \quad 7100 \\ \quad \underline{1065} \end{array}$$

$$\begin{array}{r} 16. \ 46600 \\ \quad 9320 \\ \quad \underline{1398} \end{array}$$

$$\begin{array}{r} 17. \ 57700 \\ \quad 5770 \\ \quad \underline{2308} \end{array}$$

$$\begin{array}{r} 18. \ 68800 \\ \quad 6880 \\ \quad \underline{2064} \end{array}$$

MULTIPLICATION IN GENERAL 63

19. 79900

7990

3196

20. 24600

9840

1107

21. 70200

9320

1170

Exercise No. 102

Multiplication Table Drill

Use Table II on page 48.

Multiply the numbers in Column A successively by 4, 6, 7, 8, 9, 11, 12, 13 and 14.

Repeat this exercise three times.

Exercise No. 103

Two-Column Addition

You are now ready to start adding two columns at a time. Take Exercise No. 13 on page 11. *Add from the top down.*

Two-column addition is simply an application of the left-to-right methods which you have already learned. To illustrate with the first example:

43

62

78

81

14

87

This would be added: 43, 103, 105, 175, 183, 263, 264, 274, 278, 358, 365. These are the actual steps, but with practice you will read this as 105, 183, 264, 278, 365.

Exercise No. 104

Factoring

Factor the numbers from 93 to 129 inclusive in the form shown in the table on pages 146 and 147.

64 THE ART OF CALCULATION

Exercise No. 105

Mental Addition

Read each of the following examples once, write it three times, and then add it mentally from left to right.

$$\begin{array}{r} 1. \ 112700 \\ \quad 3220 \\ \quad \underline{161} \end{array}$$

$$\begin{array}{r} 2. \ 136800 \\ \quad 5130 \\ \quad \underline{342} \end{array}$$

$$\begin{array}{r} 3. \ 162900 \\ \quad 2400 \\ \quad \underline{181} \end{array}$$

$$\begin{array}{r} 4. \ 105700 \\ \quad 1510 \\ \quad \underline{302} \end{array}$$

$$\begin{array}{r} 5. \ 128800 \\ \quad 3220 \\ \quad \underline{161} \end{array}$$

$$\begin{array}{r} 6. \ 153900 \\ \quad 5130 \\ \quad \underline{342} \end{array}$$

$$\begin{array}{r} 7. \ 151200 \\ \quad 5040 \\ \quad \underline{756} \end{array}$$

$$\begin{array}{r} 8. \ 183400 \\ \quad 7860 \\ \quad \underline{262} \end{array}$$

$$\begin{array}{r} 9. \ 176400 \\ \quad 5040 \\ \quad \underline{252} \end{array}$$

$$\begin{array}{r} 10. \ 209600 \\ \quad 7860 \\ \quad \underline{524} \end{array}$$

$$\begin{array}{r} 11. \ 104800 \\ \quad 5240 \\ \quad \underline{524} \end{array}$$

$$\begin{array}{r} 12. \ 103200 \\ \quad 6880 \\ \quad \underline{860} \end{array}$$

$$\begin{array}{r} 13. \ 114100 \\ \quad 6520 \\ \quad \underline{978} \end{array}$$

$$\begin{array}{r} 14. \ 112800 \\ \quad 7050 \\ \quad \underline{423} \end{array}$$

$$\begin{array}{r} 15. \ 126000 \\ \quad 7560 \\ \quad \underline{756} \end{array}$$

$$\begin{array}{r} 16. \ 111000 \\ \quad 9250 \\ \quad \underline{740} \end{array}$$

$$\begin{array}{r} 17. \ 104400 \\ \quad 8700 \\ \quad \underline{870} \end{array}$$

$$\begin{array}{r} 18. \ 135900 \\ \quad 9060 \\ \quad \underline{302} \end{array}$$

$$\begin{array}{r} 19. \ 112800 \\ \quad 9870 \\ \quad \underline{141} \end{array}$$

$$\begin{array}{r} 20. \ 130500 \\ \quad 8700 \\ \quad \underline{435} \end{array}$$

$$\begin{array}{r} 21. \ 136800 \\ \quad 6800 \\ \quad \underline{684} \end{array}$$

MULTIPLICATION IN GENERAL 65

Exercise No. 106

Mental Multiplication

Multiply mentally by 14 the numbers in Table I on page 7.

Exercise No. 107

Two-Column Addition

Do the examples in Exercise No. 17 on page 15 by adding two columns at a time. *Add from the bottom up.*

Exercise No. 108

Factoring

Factor the numbers from 112 to 145 inclusive in the form shown in the table on pages 146 and 147.

Exercise No. 109

Mental Addition

Read each of the following examples once, write it three times, and then add it mentally from left to right.

$$\begin{array}{r} 1. \ 121000 \\ \quad 14520 \\ \quad \quad 484 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \ 217600 \\ \quad 10880 \\ \quad \quad 544 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \ 253800 \\ \quad 14100 \\ \quad \quad 846 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \ 116000 \\ \quad 11600 \\ \quad \quad 464 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \ 145200 \\ \quad 14520 \\ \quad \quad 726 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \ 224800 \\ \quad 10880 \\ \quad \quad 816 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \ 171500 \\ \quad 24010 \\ \quad \quad 343 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \ 211800 \\ \quad 10590 \\ \quad \quad 706 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \ 344700 \\ \quad 22980 \\ \quad \quad 383 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \ 129200 \\ \quad 16150 \\ \quad \quad 323 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \ 166500 \\ \quad 19980 \\ \quad \quad 666 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \ 290400 \\ \quad 14520 \\ \quad \quad 363 \\ \hline \end{array}$$

66 THE ART OF CALCULATION

13. 335700	14. 272400	15. 324800
18650	18160	23200
<u>746</u>	<u>454</u>	<u>928</u>

16. 124200	17. 317800	18. 371200
20700	18160	23200
<u>828</u>	<u>454</u>	<u>924</u>

19. 395500	20. 210000	21. 540800
34200	36750	33800
<u>565</u>	<u>525</u>	<u>676</u>

Exercise No. 110

Written Multiplication

Multiply by 1314 the numbers in Table III on page 49.

Exercise No. 111

Two-Column Addition

Do the examples in Exercise No. 26 on page 23 by adding two columns at a time. *Add from the top down.*

Exercise No. 112

Factoring

Factor the numbers from 130 to 162 inclusive in the form shown in the table on page 147.

Exercise No. 113

Mental Addition

Read each of the following examples once, write it three times, and then add it mentally from left to right.

1. 123200	2. 187800	3. 254400
39800	37560	44520
<u>1232</u>	<u>1878</u>	<u>2544</u>

MULTIPLICATION IN GENERAL 67

$$\begin{array}{r} 4. \ 323000 \\ \ 51680 \\ \hline \ 3230 \end{array}$$

$$\begin{array}{r} 5. \ 393600 \\ \ 59040 \\ \hline \ 3936 \end{array}$$

$$\begin{array}{r} 6. \ 466200 \\ \ 26640 \\ \hline \ 4662 \end{array}$$

$$\begin{array}{r} 7. \ 616200 \\ \ 41160 \\ \hline \ 1392 \end{array}$$

$$\begin{array}{r} 8. \ 121200 \\ \ 48480 \\ \hline \ 2424 \end{array}$$

$$\begin{array}{r} 9. \ 184800 \\ \ 55440 \\ \hline \ 3080 \end{array}$$

$$\begin{array}{r} 10. \ 250400 \\ \ 25040 \\ \hline \ 3956 \end{array}$$

$$\begin{array}{r} 11. \ 318000 \\ \ 31800 \\ \hline \ 4452 \end{array}$$

$$\begin{array}{r} 12. \ 387600 \\ \ 38760 \\ \hline \ 1292 \end{array}$$

$$\begin{array}{r} 13. \ 439200 \\ \ 43920 \\ \hline \ 1312 \end{array}$$

$$\begin{array}{r} 14. \ 532800 \\ \ 53280 \\ \hline \ 1998 \end{array}$$

$$\begin{array}{r} 15. \ 608400 \\ \ 60840 \\ \hline \ 2704 \end{array}$$

$$\begin{array}{r} 16. \ 139200 \\ \ 34800 \\ \hline \ 1392 \end{array}$$

$$\begin{array}{r} 17. \ 143400 \\ \ 28680 \\ \hline \ 1434 \end{array}$$

$$\begin{array}{r} 18. \ 218700 \\ \ 36350 \\ \hline \ 2187 \end{array}$$

$$\begin{array}{r} 19. \ 294800 \\ \ 44220 \\ \hline \ 2948 \end{array}$$

$$\begin{array}{r} 20. \ 373500 \\ \ 52290 \\ \hline \ 3735 \end{array}$$

$$\begin{array}{r} 21. \ 454200 \\ \ 60560 \\ \hline \ 4542 \end{array}$$

Exercise No. 114

Continuous Addition Drill

Count by 7's to 175.

Count by 8's to 200.

Count by 9's to 225.

Count by 11's to 275.

Count by 12's to 300.

Count by 13's to 325.

68 THE ART OF CALCULATION

Count by 14's to 350.

Count by 15's to 375.

Repeat this exercise three times.

Exercise No. 115

Two-Column Addition

Do the examples in Exercise No. 34 on page 28 by adding two columns at a time. *Add from the bottom up.*

Exercise No. 116

Multiplication Table Drill

Use Table II on page 48.

Multiply the numbers in Column B successively by 6, 7, 8, 9, 11, 12, 13, 14 and 15.

Repeat this exercise three times.

Exercise No. 117

Factoring

Factor the numbers from 146 to 179 inclusive in the form shown in the table on page 147.

Exercise No. 118

Two-Column Addition

Do the examples in Exercise No. 38 on page 32 by adding two columns at a time. *Add from the top down.*

It slows up addition by two columns to keep repeating the number of hundreds as you go along. A good plan is to keep tally of the number of hundreds with a pencil. In all addition of long columns write numbers to be carried either at the head of the next column or beneath the figures in the total as you set them down. When looking for errors in addition, add in the opposite direction from that in which the addition was originally performed.

MULTIPLICATION IN GENERAL 69

Exercise No. 119

Mental Multiplication

Multiply mentally by 15 the numbers in Table I on page 7.

Exercise No. 120

Two-Column Addition

Do the examples in Exercise No. 41 on page 34 by adding two columns at a time. *Add from the bottom up.*

Exercise No. 121

Factoring

Factor the numbers from 163 to 194 inclusive in the form shown in the table on page 147.

Exercise No. 122

Two-Column Addition

Do the examples in Exercise No. 48 on page 39 by adding two columns at a time. *Add from the top down.*

Exercise No. 123

Written Multiplication

Multiply by 1415 the numbers in Table III on page 49.

Exercise No. 124

Two-Column Addition

Do the examples in Exercise No. 54 on page 41 by adding two columns at a time. *Add from the bottom up.*

70 THE ART OF CALCULATION

Exercise No. 125

Factoring

Factor the numbers from 180 to 209 inclusive in the form shown in the table on page 147.

Exercise No. 126

Two-Column Addition

Do the examples in Exercise No. 62 on page 44 by adding two columns at a time. *Add from the top down.*

Exercise No. 127

Continuous Addition Drill

Count by 8's to 200.

Count by 9's to 225.

Count by 11's to 275.

Count by 12's to 300.

Count by 13's to 325.

Count by 14's to 350.

Count by 15's to 375.

Count by 16's to 400.

Repeat this exercise three times.

Exercise No. 128

Three-Column Addition

With the practice you have had in two-column addition you should now be able to add three columns at a time. Try this with the examples in Exercise No. 38 on page 32. No additional exercises in three-column addition are given, but you can of course practice it on your own account if you so desire.

MULTIPLICATION IN GENERAL 71

Exercise No. 129

Multiplication Table Drill

Use Table II on page 48.

Multiply the numbers in Column C successively by 7, 8, 9, 11, 12, 13, 14, 15 and 16.

Repeat this exercise three times.

Exercise No. 130

Factoring

Factor the numbers from 195 to 224 inclusive in the form shown in the table on pages 147 and 148.

Exercise No. 131

Mental Multiplication

Multiply mentally by 16 the numbers in Table I on page 7.

Exercise No. 132

Written Multiplication

Multiply by 1516 the numbers in Table III on page 49.

Exercise No. 133

Factoring

Factor the numbers from 210 to 239 inclusive in the form shown in the table on pages 147 and 148.

DIVISION IN GENERAL

Division is multiplication in reverse. As you improve in multiplication you automatically develop your skill at division. For this reason it has been considered unnecessary to include any exercises in long division.

Exercises, however, are given in mental division, in order to round out your general calculating ability. These exercises are of the following types:

First you use the numbers from 2 to 25 as direct divisors, securing quotients from 1 to 99. Then you divide by the numbers from 2 to 9, finding answers of three places. Again, you divide by three-place numbers to arrive at quotients of one figure plus a remainder; the remainder is included so that the answer cannot be guessed but must be calculated accurately. Finally, you divide by numbers of two places and get results of two places. As division is somewhat more complicated, the exercises in division are not carried so far as those in multiplication.

Exercise No. 134

Mental Division

Divide mentally by 2 the answers to Exercise No. 45 as given on pages 161 and 162. Compare your answers with Table I on page 7.

Exercise No. 135

Continuous Addition Drill

Count by 9's to 225.

Count by 11's to 275.

Count by 12's to 300.

Count by 13's to 325.

Count by 14's to 350.

Count by 15's to 375.

Count by 16's to 400.

Count by 17's to 425.

Repeat this exercise three times.

Exercise No. 136

Mental Division

Divide mentally by 3 the answers to Exercise No. 46 as given on page 162. Compare your answers with Table I on page 7.

Exercise No. 137

Multiplication Table Drill

Use Table II on page 48.

Multiply mentally the numbers in Column D by 8, 9, 11, 12, 13, 14, 15, 16 and 17.

Repeat this exercise three times.

Exercise No. 138

Factoring

Factor the numbers from 225 to 254 inclusive in the form shown in the table on page 148.

Exercise No. 139

Mental Division

Divide mentally by 4 the answers to Exercise No. 47 as given on page 162. Compare your answers with Table I on page 7.

Exercise No. 140

Mental Multiplication

Multiply mentally by 17 the numbers in Table I on page 7.

Exercise No. 141**Written Multiplication**

Multiply by 1617 the numbers in Table III on page 49. Make a single multiplication of pairs of figures in the multiplicand up to 17.

Exercise No. 142**Factoring**

Factor the numbers from 240 to 269 inclusive in the form shown in the Table on page 148.

Exercise No. 143**Mental Division**

Divide mentally by 5 the answers to Exercise No. 50 as given on page 163. Compare your answers with Table I on page 7.

Exercise No. 144**Continuous Addition Drill**

Count by 11's to 275.

Count by 12's to 300.

Count by 13's to 325.

Count by 14's to 350.

Count by 15's to 375.

Count by 16's to 400.

Count by 17's to 425.

Count by 18's to 450.

Repeat this exercise three times.

Exercise No. 145**Multiplication Table Drill**

Use Table II on page 48.

Multiply mentally the numbers in Column E by 9, 11, 12, 13, 14, 15, 16, 17 and 18.

Repeat this exercise three times.

Exercise No. 146**Factoring**

Factor the numbers from 255 to 284 inclusive in the form shown in the table on page 148.

Exercise No. 147**Mental Division**

Divide mentally by 6 the answers to Exercise No. 52 as given on page 163. Compare your answers with Table I on page 7.

Exercise No. 148**Mental Multiplication**

Multiply mentally by 18 the numbers in Table I on page 7.

Exercise No. 149**Written Multiplication**

Multiply by 1718 the numbers in Table III on page 49. Make a single multiplication of pairs of figures in the multiplicand up to 18.

Exercise No. 150**Factoring**

Factor the numbers from 270 to 299 inclusive in the form shown in the table on pages 148.

Exercise No. 151**Mental Division**

Divide mentally by 7 the answers to Exercise No. 53 as given on pages 163 and 164. Compare your answers with Table I on page 7.

Exercise No. 152

Continuous Addition Drill

Count by 12's to 300.

Count by 13's to 325.

Count by 14's to 350.

Count by 15's to 375.

Count by 16's to 400.

Count by 17's to 425.

Count by 18's to 450.

Count by 19's to 475.

Repeat this exercise three times.

Exercise No. 153

Multiplication Table Drill

Use Table II on page 48.

Multiply mentally the numbers in Column F by 11, 12, 13, 14, 15, 16, 17, 18 and 19.

Repeat this exercise three times.

Exercise No. 154

Factoring

Factor the numbers from 285 to 312 inclusive in the form shown in the table on page 148.

Exercise No. 155

Mental Division

Divide mentally by 8 the answers to Exercise No. 56 as given on page 164. Compare your answers with Table I on page 7 .

Exercise No. 156

Mental Multiplication

Multiply mentally by 19 the numbers in Table I on page 7 .

Exercise No. 157**Factoring**

Factor the numbers from 300 to 328 inclusive in the form shown in the table on page 148.

Exercise No. 158**Mental Division**

Divide mentally by 9 the answers to Exercise No. 60 as given on page 164. Compare your answers with Table I on page 7 .

Exercise No. 159**Written Multiplication**

Multiply by 1819 the numbers in Table III on page 49. Make a single multiplication of pairs of figures in the multiplicand up to 19.

Exercise No. 160**Factoring**

Factor the numbers from 313 to 343 inclusive in the form shown in the table on page 149.

Exercise No. 161**Mental Division**

Divide mentally by 11 the answers to Exercise No. 61 as given on page 165. Compare your answers with Table I on page 7 .

78 THE ART OF CALCULATION

Exercise No. 162

Multiplication Table Drill

Use Table II on page 48.

Multiply mentally the numbers in Column G by 12, 13, 14, 15, 16, 17, 18, 19 and 20.

Exercise No. 163

Factoring

Factor the numbers from 329 to 359 inclusive in the form shown in the table on pages 148 and 149.

Exercise No. 164

Mental Division

Divide mentally by 12 the answers to Exercise No. 77 as given on page 166. Compare your answers with Table I on page 7.

Exercise No. 165

Mental Multiplication

Multiply mentally by 20 the numbers in Table I on page 7.

Exercise No. 166

Written Multiplication

Multiply by 1920 the numbers in Table III on page 49. Make a single multiplication of pairs of figures in the multiplicand up to 20.

Exercise No. 167

Factoring

Factor the numbers from 344 to 372 inclusive in the form shown in the table on page 149.

Exercise No. 168**Mental Division**

Divide mentally by 13 the answers to Exercise No. 90 as given on page 167. Compare your answers with Table I on page 7.

Exercise No. 169**Continuous Addition Drill**

Count by 13's to 325.

Count by 14's to 350.

Count by 15's to 375.

Count by 16's to 400.

Count by 17's to 425.

Count by 18's to 450.

Count by 19's to 475.

Count by 21's to 525.

Exercise No. 170**Multiplication Table Drill**

Use Table II on page 48.

Multiply mentally the numbers in Column H by 12, 13, 14, 15, 16, 17, 18, 19, 20 and 21.

Exercise No. 171**Factoring**

Factor the numbers from 360 to 386 inclusive in the form shown in the table on page 149.

Exercise No. 172**Mental Multiplication**

Multiply mentally by 21 the numbers in Table I on page 7.

80 THE ART OF CALCULATION

Exercise No. 173

Written Multiplication

Multiply by 2021 the numbers in Table III on page 49. Make a single multiplication of pairs of figures in the multiplicand up to 21.

Exercise No. 174

Factoring

Factor the numbers from 373 to 399 inclusive in the form shown in the table on pages 149 and 150.

Exercise No. 175

Mental Division

Divide mentally by 14 the answers to Exercise No. 106 as given on page 168. Compare your answers with Table I on page 7.

Exercise No. 176

Continuous Addition Drill

Count by 14's to 350.

Count by 15's to 375.

Count by 16's to 400.

Count by 17's to 425.

Count by 18's to 450.

Count by 19's to 475.

Count by 21's to 525.

Count by 22's to 550.

Repeat this exercise three times.

Exercise No. 177**Multiplication Table Drill**

Use Table II on page 48.

Multiply mentally the numbers in Column J by 13, 14, 15, 16, 17, 18, 19, 20, 21 and 22.

Exercise No. 178**Factoring**

Factor the numbers from 387 to 413 inclusive in the form shown in the table on pages 149 and 150.

Exercise No. 179**Mental Multiplication**

Multiply mentally by 22 the numbers in Table I on page 7.

Exercise No. 180**Written Multiplication**

Multiply by 2122 the numbers in Table III on page 49. Make a single multiplication of pairs of figures in the multiplicand up to 22.

Exercise No. 181**Factoring**

Factor the numbers from 400 to 427 inclusive in the form shown in the table on page 150.

82 THE ART OF CALCULATION

Exercise No. 182

Mental Division

Divide mentally by 15 the answers to Exercise No. 119 as given on page 169. Compare your answers with Table I on page 7.

Exercise No. 183

Continuous Addition Drill

Count by 15's to 375.

Count by 16's to 400.

Count by 17's to 425.

Count by 18's to 450.

Count by 19's to 475.

Count by 21's to 525.

Count by 22's to 550.

Count by 23's to 575.

Repeat this exercise three times.

Exercise No. 184

Multiplication Table Drill

Use Table II on page 48.

Multiply mentally the numbers in Column K by 14, 15, 16, 17, 18, 19, 20, 21, 22 and 23.

Exercise No. 185

Factoring

Factor the numbers from 414 to 440 inclusive in the form shown in the table on page 150.

Exercise No. 186

Mental Multiplication

Multiply mentally by 23 the numbers in Table I on page 7.

Exercise No. 187**Written Multiplication**

Multiply by 2223 the numbers in Table III on page 49. Make a single multiplication of pairs of figures in the multiplicand up to 23.

Exercise No. 188**Factoring**

Factor the numbers from 428 to 455 inclusive in the form shown in the table on page 150.

Exercise No. 189**Mental Division**

Divide mentally by 16 the answers to Exercise No. 131 as given on pages 169 and 170. Compare your answers with Table I on page 7 .

Exercise No. 190**Continuous Addition Drill**

Count by 16's to 400.

Count by 17's to 425.

Count by 18's to 450.

Count by 19's to 475.

Count by 21's to 525.

Count by 22's to 550.

Count by 23's to 575.

Count by 24's to 600.

Repeat this exercise three times.

Exercise No. 191**Multiplication Table Drill**

Use Table II on page 48.

Multiply mentally the numbers in Column L by 15, 16, 17, 18, 19, 20, 21, 22, 23 and 24.

84 THE ART OF CALCULATION

Exercise No. 192

Factoring

Factor the numbers from 441 to 467 inclusive in the form shown in the table on pages 150 and 151.

Exercise No. 193

Mental Multiplication

Multiply mentally by 24 the numbers in Table I on page 7 .

Exercise No. 194

Written Multiplication

Multiply by 2324 the numbers in Table III on page 49. Make a single multiplication of pairs of figures in the multiplicand up to 24.

Exercise No. 195

Factoring

Factor the numbers from 456 to 479 inclusive in the form shown in the table on pages 150 and 151.

Exercise No. 196

Mental Division

Divide mentally by 17 the answers to Exercise No. 140 as given on page 170. Compare your answers with Table I on page 7.

Exercise No. 197

Continuous Addition Drill

Count by 17's to 425.

Count by 18's to 450.

Count by 19's to 475.

Count by 21's to 525.

Count by 22's to 550.

Count by 23's to 575.

Count by 24's to 600.

Count by 25's to 625.

Repeat this exercise three times.

Exercise No. 198

Multiplication Table Drill

Use Table II on page 48.

Multiply mentally the numbers in Column M by 16, 17, 18, 19, 20, 21, 22, 23, 24 and 25.

Exercise No. 199

Factoring

Factor the numbers from 468 to 491 inclusive in the form shown in the table on page 151.

Exercise No. 200

Mental Multiplication

Multiply mentally by 25 the numbers in Table I on page 7.

Exercise No. 201

Written Multiplication

Multiply by 2425 the numbers in Table III on page 49. Make a single multiplication of pairs of figures in the multiplicand up to 25.

Exercise No. 202

Factoring

Factor the numbers from 480 to 503 inclusive in the form shown in the table on page 151.

Exercise No. 203

Mental Division

Divide mentally by 18 the answers to Exercise No. 148 as given on page 170 and 171. Compare your answers with Table I on page 7 .

Exercise No. 204

Mental Multiplication

Multiply mentally by 20 the numbers in Table I on page 7 .

Exercise No. 205

Continuous Addition Drill

Count by 18's to 450.

Count by 19's to 475.

Count by 21's to 525.

Count by 22's to 550.

Count by 23's to 575.

Count by 24's to 600.

Count by 25's to 625.

Repeat this exercise three times.

Exercise No. 206

Factoring

Factor the numbers from 492 to 515 inclusive in the form shown in the table on page 151.

Exercise No. 207

Continuous Addition Drill

Count by 19's to 475.

Count by 21's to 525.

Count by 22's to 550.

Count by 23's to 575.

Count by 24's to 600.

Count by 25's to 625.

Repeat this exercise three times.

Exercise No. 208**Mental Multiplication**

Multiply mentally by 30 the numbers in Table I on page 7.

Exercise No. 209**Factoring**

Factor the numbers from 504 to 527 inclusive in the form shown in the table on page 151.

Exercise No. 210**Mental Division**

Divide mentally by 19 the answers to Exercise No. 149 as given on page 171. Compare your answers with Table I on page 7.

Exercise No. 211**Continuous Addition Drill**

Count by 21's to 525.

Count by 22's to 550.

Count by 23's to 575.

Count by 24's to 600.

Count by 25's to 625.

Repeat this exercise three times.

Exercise No. 212**Mental Multiplication**

Multiply mentally by 40 the numbers in Table I on page 7.

Exercise No. 213**Factoring**

Factor the numbers from 516 to 539 inclusive in the form shown in the table on page 151.

88 THE ART OF CALCULATION

Exercise No. 214

Continuous Addition Drill

Count by 22's to 550.

Count by 23's to 575.

Count by 24's to 600.

Count by 25's to 625.

Repeat this exercise three times.

Exercise No. 215

Mental Multiplication

Multiply mentally by 50 the numbers in Table I on page 7.

Exercise No. 216

Factoring

Factor the numbers from 528 to 551 inclusive in the form shown in the table on pages 151 and 152.

Exercise No. 217

Continuous Addition Drill

Count by 23's to 575.

Count by 24's to 600.

Count by 25's to 625.

Repeat this exercise three times.

Exercise No. 218

Mental Division

Divide mentally by 20 the answers to Exercise No. 165 as given on page 172. Compare your answers with Table I on page 7.

Exercise No. 219**Mental Multiplication**

Multiply mentally by 60 the numbers in Table I on page 7.

Exercise No. 220**Factoring**

Factor the numbers from 540 to 564 inclusive in the form shown in the table on page 152.

Exercise No. 221**Continuous Addition Drill**

Count by 24's to 600.

Count by 25's to 625.

Repeat this exercise three times.

Exercise No. 222**Mental Multiplication**

Multiply mentally by 70 the numbers in Table I on page 7.

Exercise No. 223**Factoring**

Factor the numbers from 552 to 576 inclusive in the form shown in the table on page 152.

Exercise No. 224**Mental Division**

Divide mentally by 21 the answers to Exercise No. 172 as given on page 172. Compare your answers with Table I on page 7.

Exercise No. 225**Continuous Addition Drill**

Count by 25's to 625.

Repeat this exercise three times.

Exercise No. 226**Mental Multiplication**

Multiply mentally by 80 the numbers in Table I on page 7.

Exercise No. 227**Factoring**

Factor the numbers from 565 to 592 inclusive in the form shown in the table on page 152.

Exercise No. 228**Mental Multiplication**

Multiply mentally by 90 the numbers in Table I on page 7.

Exercise No. 229**Multiplying Three Figures by One**

We are now ready to start the mental multiplication of numbers of three places by numbers of one place. Work from left to right. Immediately name the first partial product as hundreds or thousands. Thus, taking the fourth example, this would be calculated as 800, 900, 902. The fifth example would be figured as 1000, 1120, 1124.

When dealing with numbers in the thousands be sure to consider the thousands as such and not as so many hundreds. If you wish, however, you may shorten the terminology. You may, for instance, think of one thousand one

hundred twenty-six simply as one, one twenty-six, or as one, one two six.

Perform mentally the following multiplications.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 121×2 | 8. 842×2 | 15. 663×2 |
| 2. 232×2 | 9. 953×2 | 16. 721×2 |
| 3. 343×2 | 10. 161×2 | 17. 832×2 |
| 4. 451×2 | 11. 222×2 | 18. 943×2 |
| 5. 562×2 | 12. 333×2 | 19. 151×2 |
| 6. 623×2 | 13. 441×2 | 20. 262×2 |
| 7. 731×2 | 14. 552×2 | |

Exercise No. 230

Factoring

Factor the numbers from 577 to 605 inclusive in the form shown in the table on page 152.

Exercise No. 231

Mental Division

Divide mentally by 22 the answers to Exercise No. 179 as given on page 173. Compare your answers with Table I on page 7.

Exercise No. 232

Mental Multiplication

Multiply mentally by 110 the numbers in Table I on page 7.

Exercise No. 233

Multiplying Three Figures by One

Perform mentally the following multiplications.

- | | | |
|-------------------|-------------------|-------------------|
| 1. 131×3 | 3. 353×3 | 5. 571×3 |
| 2. 242×3 | 4. 464×3 | 6. 632×3 |

92 THE ART OF CALCULATION

- | | | |
|--------------------|--------------------|--------------------|
| 7. 743×3 | 12. 344×3 | 17. 841×3 |
| 8. 854×3 | 13. 451×3 | 18. 952×3 |
| 9. 961×3 | 14. 562×3 | 19. 163×3 |
| 10. 172×3 | 15. 673×3 | 20. 274×3 |
| 11. 233×3 | 16. 734×3 | |

Exercise No. 234

Factoring

Factor the numbers from 593 to 625 inclusive in the form shown in the table on pages 152 and 153.

Exercise No. 235

Mental Division

Divide mentally by 23 the answers to Exercise No. 186 as given on pages 173 and 174. Compare your answers with Table I on page 7.

Exercise No. 236

Mental Multiplication

Multiply mentally by 120 the numbers in Table I on page 7.

Exercise No. 237

Multiplying Three Figures by One

Perform mentally the following multiplications.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 141×4 | 8. 863×4 | 15. 685×4 |
| 2. 252×4 | 9. 974×4 | 16. 741×4 |
| 3. 363×4 | 10. 185×4 | 17. 852×4 |
| 4. 474×4 | 11. 241×4 | 18. 963×4 |
| 5. 585×4 | 12. 352×4 | 19. 174×4 |
| 6. 641×4 | 13. 463×4 | 20. 285×4 |
| 7. 752×4 | 14. 574×4 | |

Exercise No. 238**Mental Division**

Divide mentally by 24 the answers to Exercise No. 193 as given on page 174. Compare your answers with Table I on page 7.

Exercise No. 239**Mental Multiplication**

Multiply mentally by 130 the numbers in Table I on page 7.

Exercise No. 240**Multiplying Three Figures by One**

Perform mentally the following multiplications.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 151×5 | 8. 872×5 | 15. 693×5 |
| 2. 262×5 | 9. 983×5 | 16. 754×5 |
| 3. 373×5 | 10. 194×5 | 17. 865×5 |
| 4. 484×5 | 11. 255×5 | 18. 976×5 |
| 5. 595×5 | 12. 366×5 | 19. 181×5 |
| 6. 656×5 | 13. 471×5 | 20. 292×5 |
| 7. 761×5 | 14. 582×5 | |

Exercise No. 241**Mental Division**

Divide mentally by 25 the answers to Exercise No. 200 as given on pages 174 and 175. Compare your answers with Table I on page 7.

Exercise No. 242**Mental Multiplication**

Multiply mentally by 140 the numbers in Table I on page 7.

94 THE ART OF CALCULATION

Exercise No. 243

Multiplying Three Figures by One

Perform mentally the following multiplications.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 141×6 | 8. 851×6 | 15. 661×6 |
| 2. 252×6 | 9. 962×6 | 16. 772×6 |
| 3. 363×6 | 10. 173×6 | 17. 883×6 |
| 4. 474×6 | 11. 284×6 | 18. 994×6 |
| 5. 585×6 | 12. 395×6 | 19. 145×6 |
| 6. 696×6 | 13. 446×6 | 20. 256×6 |
| 7. 747×6 | 14. 557×6 | |

Exercise No. 244

Mental Multiplication

Multiply mentally by 150 the numbers in Table I on page 7.

Exercise No. 245

Multiplying Three Figures by One

Perform mentally the following multiplications.

- | | | |
|-------------------|--------------------|-----------------------------|
| 1. 131×7 | 8. 838×7 | 15. $\cancel{637} \times 7$ |
| 2. 242×7 | 9. 941×7 | 16. 748×7 |
| 3. 353×7 | 10. 152×7 | 17. 851×7 |
| 4. 464×7 | 11. 263×7 | 18. 962×7 |
| 5. 575×7 | 12. 374×7 | 19. $\cancel{173} \times 7$ |
| 6. 686×7 | 13. 485×7 | 20. 284×7 |
| 7. 797×7 | 14. 596×7 | |

Exercise No. 246

Mental Multiplication

Multiply mentally by 160 the numbers in Table I on page 7.

Exercise No. 247**Multiplying Three Figures by One**

Perform mentally the following multiplications.

- | | | |
|-------------------|--|--|
| 1. 141×8 | 8. 858×8 | 15. 666×8 |
| 2. 252×8 | 9. 969×8 | 16. 777×8 |
| 3. 363×8 | 10. 171×8 | 17. 888×8 |
| 4. 474×8 | 11. 282×8 | 18. 999×8 |
| 5. 585×8 | 12. 393×8 | 19. 741×8 |
| 6. 696×8 | 13. 444×8 | 20. 652×8 |
| 7. 747×8 | 14. 555×8 | |

FRACTIONS IN GENERAL

The multiplication or the division of fractions will present no difficulty to the student of these pages since it is simply a matter of combining operations in which he is well practised.

What needs more particular attention is the addition and subtraction of the kinds of fractions most commonly encountered in practical work in office, shop and home. The average person would immediately reach for a pencil if asked the sum of $\frac{3}{4}$ and $\frac{5}{8}$ or the difference between $1\frac{1}{3}$ and $\frac{3}{8}$. Yet a little practice with calculations of this kind makes it very easy to perform them mentally.

The succeeding examples in addition and subtraction of fractions are based on the possible combinations of two fractions of the orders of halves, quarters, eighths, sixteenths, thirds, sixths, twelfths, fifths and tenths.

These exercises are to stimulate memory and rapid thinking. No instructions are given as to how to perform them because it is assumed that the student is familiar with the reduction of fractions to a common denominator.

Exercise No. 248

Reduction of Fractions

1. Reduce to eighths: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$
2. Reduce to sixteenths: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$
3. Reduce to sixths: $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$
4. Reduce to twelfths: $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$
5. Reduce to twenty-fourths: $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{5}{12}$, $\frac{1}{2}$, $\frac{7}{12}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $1\frac{1}{3}$
6. Reduce to tenths: $\frac{1}{5}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{5}$

7. Reduce to twentieths: $\frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}$

8. Reduce to fortieths: $\frac{1}{10}, \frac{1}{5}, \frac{1}{3}, \frac{1}{4}, \frac{3}{10}, \frac{2}{5}, \frac{2}{3}, \frac{1}{2}, \frac{3}{5}, \frac{5}{8}, \frac{7}{10}, \frac{1}{2}, \frac{4}{5}, \frac{7}{8}, \frac{9}{10}$

9. Reduce to fifteenths: $\frac{1}{5}, \frac{1}{3}, \frac{2}{5}, \frac{3}{5}, \frac{2}{3}, \frac{4}{5}$

10. Reduce to thirtieths: $\frac{1}{10}, \frac{1}{6}, \frac{1}{5}, \frac{3}{10}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{7}{10}, \frac{1}{3}, \frac{5}{6}, \frac{9}{10}$

Exercise No. 249

Mental Multiplication

Multiply mentally by 170 the numbers in Table I on page 7.

Exercise No. 250

Addition of Fractions

Add the following mentally.

- | | | | |
|---------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| 1. $\frac{1}{2} + \frac{1}{4}$ | 11. $\frac{3}{4} + \frac{1}{8}$ | 21. $\frac{1}{2} + \frac{13}{16}$ | 31. $\frac{3}{4} + \frac{1}{16}$ |
| 2. $\frac{1}{2} + \frac{3}{4}$ | 12. $\frac{3}{4} + \frac{3}{8}$ | 22. $\frac{1}{2} + \frac{15}{16}$ | 32. $\frac{3}{4} + \frac{3}{16}$ |
| 3. $\frac{1}{2} + \frac{1}{8}$ | 13. $\frac{3}{4} + \frac{5}{8}$ | 23. $\frac{1}{4} + \frac{1}{16}$ | 33. $\frac{3}{4} + \frac{5}{16}$ |
| 4. $\frac{1}{2} + \frac{3}{8}$ | 14. $\frac{3}{4} + \frac{7}{8}$ | 24. $\frac{1}{4} + \frac{3}{16}$ | 34. $\frac{3}{4} + \frac{7}{16}$ |
| 5. $\frac{1}{2} + \frac{5}{8}$ | 15. $\frac{1}{2} + \frac{1}{16}$ | 25. $\frac{1}{4} + \frac{5}{16}$ | 35. $\frac{3}{4} + \frac{9}{16}$ |
| 6. $\frac{1}{2} + \frac{7}{8}$ | 16. $\frac{1}{2} + \frac{3}{16}$ | 26. $\frac{1}{4} + \frac{7}{16}$ | 36. $\frac{3}{4} + \frac{11}{16}$ |
| 7. $\frac{1}{4} + \frac{1}{8}$ | 17. $\frac{1}{2} + \frac{5}{16}$ | 27. $\frac{1}{4} + \frac{9}{16}$ | 37. $\frac{3}{4} + \frac{13}{16}$ |
| 8. $\frac{1}{4} + \frac{3}{8}$ | 18. $\frac{1}{2} + \frac{7}{16}$ | 28. $\frac{1}{4} + \frac{11}{16}$ | 38. $\frac{3}{4} + \frac{15}{16}$ |
| 9. $\frac{1}{4} + \frac{5}{8}$ | 19. $\frac{1}{2} + \frac{9}{16}$ | 29. $\frac{1}{4} + \frac{13}{16}$ | 39. $\frac{1}{8} + \frac{1}{16}$ |
| 10. $\frac{1}{4} + \frac{7}{8}$ | 20. $\frac{1}{2} + \frac{11}{16}$ | 30. $\frac{1}{4} + \frac{15}{16}$ | 40. $\frac{1}{8} + \frac{3}{16}$ |

Exercise No. 251

Multiplying Three Figures by One

- | | | |
|-------------------|--------------------|--------------------|
| 1. 152×9 | 8. 869×9 | 15. 679×9 |
| 2. 263×9 | 9. 973×9 | 16. 784×9 |
| 3. 374×9 | 10. 184×9 | 17. 895×9 |
| 4. 485×9 | 11. 295×9 | 18. 946×9 |
| 5. 596×9 | 12. 346×9 | 19. 157×9 |
| 6. 647×9 | 13. 457×9 | 20. 268×9 |
| 7. 758×9 | 14. 568×9 | |

98 THE ART OF CALCULATION

Exercise No. 252

Mental Division

Divide mentally by 2 the answers to Exercise No. 229 as given on page 175.

Exercise No. 253

Addition of Fractions

Do the last thirty examples in Exercise No. 250 on the preceding page, and also add the following.

- | | | | |
|---------------------------------|----------------------------------|---------------------------------|----------------------------------|
| 1. $\frac{1}{8} + \frac{5}{16}$ | 4. $\frac{1}{8} + \frac{11}{16}$ | 7. $\frac{3}{8} + \frac{1}{16}$ | 10. $\frac{3}{8} + \frac{7}{16}$ |
| 2. $\frac{1}{8} + \frac{7}{16}$ | 5. $\frac{1}{8} + \frac{13}{16}$ | 8. $\frac{3}{8} + \frac{3}{16}$ | |
| 3. $\frac{1}{8} + \frac{9}{16}$ | 6. $\frac{1}{8} + \frac{15}{16}$ | 9. $\frac{3}{8} + \frac{5}{16}$ | |

Exercise No. 254

Mental Multiplication

Multiply mentally by 180 the numbers in Table I on page 7.

Exercise No. 255

Mental Division

Divide mentally by 3 the answers to Exercise No. 233 as given on page 175. Compare your answers with Exercise No. 233.

Exercise No. 256

Addition of Fractions

Review the last twenty examples in Exercise No. 250 on page 97 and those in Exercise No. 253 on page 98. Also add the following.

- | | | | |
|----------------------------------|----------------------------------|---------------------------------|-----------------------------------|
| 1. $\frac{3}{8} + \frac{9}{16}$ | 4. $\frac{3}{8} + \frac{15}{16}$ | 7. $\frac{5}{8} + \frac{5}{16}$ | 10. $\frac{5}{8} + \frac{11}{16}$ |
| 2. $\frac{3}{8} + \frac{11}{16}$ | 5. $\frac{5}{8} + \frac{1}{16}$ | 8. $\frac{5}{8} + \frac{7}{16}$ | |
| 3. $\frac{3}{8} + \frac{13}{16}$ | 6. $\frac{5}{8} + \frac{5}{16}$ | 9. $\frac{5}{8} + \frac{9}{16}$ | |

Exercise No. 257**Mental Multiplication**

Multiply mentally by 190 the numbers in Table I on page 7.

Exercise No. 258**Mental Division**

Divide mentally by 4 the answers to Exercise No. 237 as given on page 175.

Exercise No. 259**Addition of Fractions**

Review the last ten examples in Exercise No. 250 on page 97, as well as those in Exercise No. 253 on page 98 and Exercise No. 256 on page 98. Also add the following.

- | | | | |
|----------------------------------|---------------------------------|----------------------------------|-----------------------------------|
| 1. $\frac{5}{8} + \frac{13}{16}$ | 4. $\frac{7}{8} + \frac{3}{16}$ | 7. $\frac{7}{8} + \frac{9}{16}$ | 10. $\frac{7}{8} + \frac{15}{16}$ |
| 2. $\frac{5}{8} + \frac{15}{16}$ | 5. $\frac{7}{8} + \frac{5}{16}$ | 8. $\frac{7}{8} + \frac{11}{16}$ | |
| 3. $\frac{7}{8} + \frac{1}{16}$ | 6. $\frac{7}{8} + \frac{7}{16}$ | 9. $\frac{7}{8} + \frac{13}{16}$ | |

Exercise No. 260**Mental Multiplication**

Multiply mentally by 200 the numbers in Table I on page 7.

Exercise No. 261**Addition of Fractions**

Review the examples in Exercise No. 253 on page 98, No. 256 on page 98 and No. 259 above. Also add the following.

- | | | | |
|---------------------------------|----------------------------------|---------------------------------|-----------------------------------|
| 1. $\frac{1}{3} + \frac{1}{6}$ | 4. $\frac{1}{3} + \frac{5}{12}$ | 7. $\frac{2}{3} + \frac{1}{12}$ | 10. $\frac{2}{3} + \frac{11}{12}$ |
| 2. $\frac{2}{3} + \frac{1}{6}$ | 5. $\frac{1}{3} + \frac{7}{12}$ | 8. $\frac{2}{3} + \frac{5}{12}$ | |
| 3. $\frac{1}{3} + \frac{1}{12}$ | 6. $\frac{1}{3} + \frac{11}{12}$ | 9. $\frac{2}{3} + \frac{7}{12}$ | |

100 THE ART OF CALCULATION

Exercise No. 262

Mental Division

Divide mentally by 5 the answers to Exercise No. 240 as given on page 175.

Exercise No. 263

Subtraction of Fractions

Perform mentally the following subtractions.

- | | | | |
|---------------------------------|----------------------------------|-----------------------------------|-----------------------------------|
| 1. $\frac{3}{4} - \frac{1}{2}$ | 8. $\frac{5}{8} - \frac{1}{2}$ | 16. $\frac{11}{16} - \frac{1}{2}$ | 24. $\frac{7}{16} - \frac{1}{4}$ |
| 2. $1\frac{1}{4} - \frac{1}{2}$ | 9. $\frac{7}{8} - \frac{1}{2}$ | 17. $\frac{13}{16} - \frac{1}{2}$ | 25. $\frac{9}{16} - \frac{1}{4}$ |
| 3. $\frac{5}{8} - \frac{1}{2}$ | 10. $1\frac{1}{8} - \frac{1}{4}$ | 18. $\frac{15}{16} - \frac{1}{2}$ | 26. $\frac{11}{16} - \frac{1}{4}$ |
| 4. $\frac{7}{8} - \frac{1}{2}$ | 11. $\frac{7}{8} - \frac{3}{4}$ | 19. $1\frac{1}{16} - \frac{1}{2}$ | 27. $\frac{13}{16} - \frac{1}{4}$ |
| 5. $1\frac{1}{8} - \frac{1}{2}$ | 12. $1\frac{1}{8} - \frac{3}{4}$ | 20. $1\frac{3}{16} - \frac{1}{2}$ | 28. $\frac{15}{16} - \frac{1}{4}$ |
| 6. $1\frac{3}{8} - \frac{1}{2}$ | 13. $1\frac{3}{8} - \frac{3}{4}$ | 21. $1\frac{5}{16} - \frac{1}{2}$ | 29. $1\frac{1}{16} - \frac{1}{4}$ |
| 7. $\frac{3}{8} - \frac{1}{4}$ | 14. $1\frac{5}{8} - \frac{3}{4}$ | 22. $1\frac{7}{16} - \frac{1}{2}$ | 30. $1\frac{3}{16} - \frac{1}{4}$ |
| | 15. $\frac{9}{16} - \frac{1}{2}$ | 23. $\frac{5}{16} - \frac{1}{4}$ | |

Exercise No. 264

Mental Multiplication

Multiply mentally by 210 the numbers in Table I on page 7.

Exercise No. 265

Subtraction of Fractions

Review the last twenty examples in Exercise No. 263 above, and also perform the following subtractions.

- | | | | |
|----------------------------------|----------------------------------|-----------------------------------|----------------------------------|
| 1. $\frac{13}{16} - \frac{3}{4}$ | 4. $1\frac{3}{16} - \frac{3}{4}$ | 7. $1\frac{9}{16} - \frac{3}{4}$ | 10. $\frac{5}{16} - \frac{1}{8}$ |
| 2. $\frac{15}{16} - \frac{3}{4}$ | 5. $1\frac{5}{16} - \frac{3}{4}$ | 8. $1\frac{11}{16} - \frac{3}{4}$ | |
| 3. $1\frac{1}{16} - \frac{3}{4}$ | 6. $1\frac{7}{16} - \frac{3}{4}$ | 9. $\frac{3}{16} - \frac{1}{8}$ | |

Exercise No. 266**Mental Division**

Divide mentally by 6 the answers to Exercise No. 243 as given on page 175.

Exercise No. 267**Addition of Fractions**

Review the examples in Exercise No. 256 on page 98, No. 259 on page 99 and No. 261 on page 99. Also perform the following additions.

- | | | | |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1. $\frac{1}{6} + \frac{1}{12}$ | 4. $\frac{1}{6} + \frac{1}{12}$ | 7. $\frac{5}{8} + \frac{7}{12}$ | 10. $\frac{1}{2} + \frac{2}{3}$ |
| 2. $\frac{1}{6} + \frac{5}{12}$ | 5. $\frac{5}{8} + \frac{1}{12}$ | 8. $\frac{5}{8} + \frac{1}{12}$ | |
| 3. $\frac{1}{6} + \frac{7}{12}$ | 6. $\frac{5}{8} + \frac{5}{12}$ | 9. $\frac{1}{2} + \frac{1}{3}$ | |

Exercise No. 268**Mental Multiplication**

Multiply mentally by 220 the numbers in Table I on page 7.

Exercise No. 269**Subtraction of Fractions**

Review the last ten examples in Exercise No. 263 on page 100 and No. 265 on page 100. Also perform the following subtractions.

- | | | | |
|----------------------------------|----------------------------------|----------------------------------|-----------------------------------|
| 1. $\frac{7}{16} - \frac{1}{8}$ | 4. $\frac{13}{16} - \frac{1}{8}$ | 7. $\frac{7}{16} - \frac{3}{8}$ | 10. $\frac{13}{16} - \frac{3}{8}$ |
| 2. $\frac{9}{16} - \frac{1}{8}$ | 5. $\frac{15}{16} - \frac{1}{8}$ | 8. $\frac{9}{16} - \frac{3}{8}$ | |
| 3. $\frac{11}{16} - \frac{1}{8}$ | 6. $1\frac{1}{16} - \frac{1}{8}$ | 9. $\frac{11}{16} - \frac{3}{8}$ | |

Exercise No. 270**Mental Division**

Divide mentally by 7 the answers to Exercise No. 245 as given on page 176.

102 THE ART OF CALCULATION

Exercise No. 271

Addition of Fractions

Review the examples in Exercise No. 259 on page 99 , No. 261 on page 99 and No. 267 on page 101. Also perform the following additions.

- | | | | |
|--------------------------------|--------------------------------|--------------------------------|---------------------------------|
| 1. $\frac{1}{2} + \frac{1}{6}$ | 4. $\frac{1}{2} + \frac{5}{6}$ | 7. $\frac{1}{3} + \frac{1}{6}$ | 10. $\frac{7}{8} + \frac{1}{8}$ |
| 2. $\frac{1}{2} + \frac{5}{6}$ | 5. $\frac{3}{4} + \frac{1}{6}$ | 8. $\frac{3}{8} + \frac{1}{6}$ | |
| 3. $\frac{1}{4} + \frac{1}{6}$ | 6. $\frac{3}{4} + \frac{5}{6}$ | 9. $\frac{5}{8} + \frac{1}{6}$ | |

Exercise No. 272

Mental Multiplication

Multiply mentally by 230 the numbers in Table I on page 7 .

Exercise No. 273

Subtraction of Fractions

Review the examples in Exercise No. 265 on page 100 and No. 269 on page 101. Also perform the following subtractions.

- | | | | |
|----------------------------------|----------------------------------|----------------------------------|-----------------------------------|
| 1. $\frac{15}{16} - \frac{2}{3}$ | 4. $1\frac{5}{16} - \frac{2}{3}$ | 7. $\frac{15}{16} - \frac{5}{8}$ | 10. $1\frac{5}{16} - \frac{5}{8}$ |
| 2. $1\frac{1}{16} - \frac{2}{3}$ | 5. $\frac{11}{16} - \frac{5}{8}$ | 8. $1\frac{1}{16} - \frac{5}{8}$ | |
| 3. $1\frac{2}{16} - \frac{2}{3}$ | 6. $\frac{12}{16} - \frac{5}{8}$ | 9. $1\frac{3}{16} - \frac{5}{8}$ | |

Exercise No. 274

Mental Division

Divide mentally by 8 the answers to Exercise No. 247 as given on page 176.

Exercise No. 275

Addition of Fractions

Review the examples in Exercise No. 261 on page 99 , No. 267 on page 101 and No. 271 on this page. Also perform the following additions.

- | | | | |
|--------------------------------|---------------------------------|---------------------------------|----------------------------------|
| 1. $\frac{1}{8} + \frac{5}{8}$ | 4. $\frac{7}{8} + \frac{5}{8}$ | 7. $\frac{1}{2} + \frac{7}{12}$ | 10. $\frac{1}{4} + \frac{5}{12}$ |
| 2. $\frac{3}{8} + \frac{5}{8}$ | 5. $\frac{1}{2} + \frac{1}{12}$ | 8. $\frac{1}{2} + \frac{1}{12}$ | |
| 3. $\frac{5}{8} + \frac{5}{8}$ | 6. $\frac{1}{2} + \frac{5}{12}$ | 9. $\frac{1}{4} + \frac{1}{12}$ | |

Exercise No. 276**Mental Multiplication**

Multiply mentally by 240 the numbers in Table I on page 7.

Exercise No. 277**Subtraction of Fractions**

Review the examples in Exercise No. 269 on page 101 and No. 273 on page 102. Also perform the following.

- | | | | |
|-----------------------------------|----------------------------------|-----------------------------------|------------------------------------|
| 1. $1\frac{7}{16} - \frac{5}{8}$ | 4. $1\frac{1}{16} - \frac{7}{8}$ | 7. $1\frac{7}{16} - \frac{7}{8}$ | 10. $1\frac{13}{16} - \frac{7}{8}$ |
| 2. $1\frac{9}{16} - \frac{5}{8}$ | 5. $1\frac{3}{16} - \frac{7}{8}$ | 8. $1\frac{9}{16} - \frac{7}{8}$ | |
| 3. $1\frac{15}{16} - \frac{7}{8}$ | 6. $1\frac{5}{16} - \frac{7}{8}$ | 9. $1\frac{11}{16} - \frac{7}{8}$ | |

Exercise No. 278**Mental Division**

Divide mentally by 9 the answers to Exercise No. 251 as given on page 176.

Exercise No. 279**Addition of Fractions**

Review the examples in Exercise No. 267 on page 101, No. 271 on page 102 and No. 275 on this page. Also perform the following additions.

- | | | | |
|---------------------------------|----------------------------------|---------------------------------|----------------------------------|
| 1. $\frac{1}{4} + \frac{7}{12}$ | 4. $\frac{3}{4} + \frac{5}{12}$ | 7. $\frac{1}{8} + \frac{1}{12}$ | 10. $\frac{1}{8} + \frac{1}{12}$ |
| 2. $\frac{1}{4} + \frac{1}{12}$ | 5. $\frac{3}{4} + \frac{7}{12}$ | 8. $\frac{1}{8} + \frac{5}{12}$ | |
| 3. $\frac{3}{4} + \frac{1}{12}$ | 6. $\frac{3}{4} + \frac{11}{12}$ | 9. $\frac{1}{8} + \frac{7}{12}$ | |

104 THE ART OF CALCULATION

Exercise No. 280

Mental Multiplication

Multiply mentally by 250 the numbers in Table I on page 7.

Exercise No. 281

Subtraction of Fractions

Review the examples in Exercise No. 273 on page 102 and No. 277 on page 103. Also perform the following subtractions.

- | | | | |
|---------------------------------|---------------------------------|----------------------------------|----------------------------------|
| 1. $\frac{1}{2} - \frac{1}{3}$ | 4. $\frac{3}{4} - \frac{1}{3}$ | 7. $\frac{3}{4} - \frac{2}{3}$ | 10. $1\frac{7}{8} - \frac{2}{3}$ |
| 2. $\frac{5}{8} - \frac{2}{3}$ | 5. $1\frac{1}{2} - \frac{1}{3}$ | 8. $1\frac{1}{12} - \frac{2}{3}$ | |
| 3. $\frac{5}{12} - \frac{1}{3}$ | 6. $1\frac{1}{4} - \frac{1}{3}$ | 9. $1\frac{1}{4} - \frac{2}{3}$ | |

Exercise No. 282

Mental Division

Divide mentally the following. Express remainders as such instead of as fractions.

- | | | |
|--------------------|---------------------|---------------------|
| 1. $328 \div 121$ | 8. $1786 \div 842$ | 15. $1998 \div 571$ |
| 2. $593 \div 232$ | 9. $2114 \div 953$ | 16. $690 \div 141$ |
| 3. $794 \div 343$ | 10. $439 \div 161$ | 17. $1208 \div 252$ |
| 4. $1249 \div 451$ | 11. $406 \div 131$ | 18. $1704 \div 363$ |
| 5. $1580 \div 562$ | 12. $776 \div 242$ | 19. $2178 \div 474$ |
| 6. $1835 \div 623$ | 13. $1164 \div 353$ | 20. $2620 \div 585$ |
| 7. $1774 \div 731$ | 14. $1574 \div 464$ | |

Exercise No. 283

Addition of Fractions

Review the examples in Exercise No. 271 on page 102, No. 275 on page 103 and No. 279 on page 103. Also perform the following additions.

- | | | | |
|---------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 1. $\frac{2}{3} + \frac{1}{12}$ | 4. $\frac{3}{8} + 1\frac{1}{12}$ | 7. $\frac{5}{8} + 1\frac{7}{12}$ | 10. $\frac{7}{8} + \frac{5}{12}$ |
| 2. $\frac{2}{3} + \frac{5}{12}$ | 5. $\frac{5}{8} + 1\frac{1}{12}$ | 8. $\frac{5}{8} + 1\frac{1}{12}$ | |
| 3. $\frac{2}{3} + \frac{7}{12}$ | 6. $\frac{5}{8} + \frac{5}{12}$ | 9. $\frac{7}{8} + 1\frac{1}{12}$ | |

Exercise No. 284

Multiplying Two Figures by Two

With this exercise we start the general multiplication of two numbers of two places each. You have had some experience with such numbers in using the numbers up to 25 as direct multipliers. In the succeeding exercises, however, the multipliers are greater than 25 and the operation is performed differently.

Multiply the whole of the multiplicand by the first figure of the multiplier; next multiply the whole of the multiplicand by the second figure of the multiplier; and finally add the two partial products.

When you multiply the first figure of the multiplicand by the first figure of the multiplier you will get a number of either three places, as in the first example (where 20×40 produces 800), or four places, as in the second example (where 2×5 produces 10). Add to this first result as you work along from left to right. Similarly, when you multiply the first figure of the multiplicand by the second figure of the multiplier, you will get a number of either two or three places.

Repeat to yourself the original example and the partial products as often as you find necessary. The need for such repetitions will grow less as you become more practised.

Taking the first example: repeat, 41×26 , 41×26 , 41×26 . 40×20 is 800, 1×2 is 2, 820. (say 1×2 rather than 1×20 because the former method is simpler when dealing with large numbers. When you think of the 2 as following the 8 it of course becomes a 20 in the product.) Repeat 820, 820, 820. 40×6 is 240, 1×6 is 6, 246. Repeat $820 + 246$, $820 + 246$, $820 + 246$. Add: 1020, 1060, 1066.

The second example is performed: 1000, 1020; 350, 357. $1020 + 357$, 1320, 1370, 1377.

106 THE ART OF CALCULATION

Most of the examples in this exercise are very simple and there can be no objection to your shortening the method given, which is a general method applicable to increasingly larger numbers. Thus in the examples illustrated you should be able to note at a glance that the first partial products are 820 and 1020.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 41×26 | 8. 41×34 | 15. 41×33 |
| 2. 51×27 | 9. 51×26 | 16. 51×34 |
| 3. 61×28 | 10. 61×27 | 17. 61×26 |
| 4. 71×29 | 11. 71×28 | 18. 71×27 |
| 5. 81×31 | 12. 81×29 | 19. 81×28 |
| 6. 91×32 | 13. 91×31 | 20. 91×29 |
| 7. 31×33 | 14. 31×32 | |

Exercise No. 285

Subtraction of Fractions

Review the examples in Exercise No. 277 on page 103 and No. 281 on page 104. Also perform the following subtractions.

- | | | | |
|--------------------------------|---------------------------------|---------------------------------|----------------------------------|
| 1. $\frac{1}{2} - \frac{1}{6}$ | 4. $1\frac{1}{2} - \frac{1}{6}$ | 7. $1\frac{5}{2} - \frac{5}{6}$ | 10. $1\frac{1}{6} - \frac{1}{2}$ |
| 2. $\frac{7}{2} - \frac{1}{6}$ | 5. $1\frac{1}{2} - \frac{5}{6}$ | 8. $1\frac{3}{2} - \frac{5}{6}$ | |
| 3. $\frac{3}{4} - \frac{1}{6}$ | 6. $1\frac{1}{4} - \frac{5}{6}$ | 9. $\frac{5}{6} - \frac{1}{2}$ | |

Exercise No. 286

Mental Division

Divide mentally the following.

- | | | |
|--------------------|--------------------|---------------------|
| 1. $445 \div 222$ | 6. $2274 \div 632$ | 11. $2830 \div 641$ |
| 2. $695 \div 333$ | 7. $2747 \div 743$ | 12. $3233 \div 752$ |
| 3. $1258 \div 441$ | 8. $3242 \div 854$ | 13. $3624 \div 863$ |
| 4. $1655 \div 552$ | 9. $3747 \div 961$ | 14. $3989 \div 974$ |
| 5. $1700 \div 663$ | 10. $533 \div 172$ | 15. $902 \div 185$ |

- | | | |
|---------------------|---------------------|---------------------|
| 16. $845 \div 151$ | 18. $2013 \div 373$ | 20. $3094 \div 595$ |
| 17. $1440 \div 262$ | 19. $2564 \div 484$ | |

Exercise No. 287

Addition of Fractions

Review the examples in Exercise No. 275 on page 103, No. 279 on page 103 and No. 283 on page 104. Also perform the following additions.

- | | | | |
|----------------------------------|---------------------------------|---------------------------------|----------------------------------|
| 1. $\frac{7}{8} + \frac{7}{12}$ | 4. $\frac{1}{3} + \frac{3}{10}$ | 7. $\frac{2}{3} + \frac{1}{10}$ | 10. $\frac{2}{3} + \frac{2}{10}$ |
| 2. $\frac{7}{8} + \frac{11}{12}$ | 5. $\frac{1}{3} + \frac{7}{10}$ | 8. $\frac{2}{3} + \frac{3}{10}$ | |
| 3. $\frac{1}{3} + \frac{1}{10}$ | 6. $\frac{1}{3} + \frac{2}{10}$ | 9. $\frac{2}{3} + \frac{7}{10}$ | |

Exercise No. 288

Multiplying Two Figures by Two

In doing exercises of this type always use the second number as the multiplier. Using the first example to illustrate, find 30 times 42 and then 5 times 42; do not work the other way around by finding 40 times 35 and then 2 times 35. This caution is given because of the special way in which the exercises are graded.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 42×35 | 8. 42×43 | 15. 42×42 |
| 2. 52×36 | 9. 52×35 | 16. 52×43 |
| 3. 62×37 | 10. 62×36 | 17. 62×34 |
| 4. 72×38 | 11. 72×37 | 18. 72×35 |
| 5. 82×39 | 12. 82×38 | 19. 82×36 |
| 6. 92×41 | 13. 92×39 | 20. 92×37 |
| 7. 32×42 | 14. 32×41 | |

Exercise No. 289

Subtraction of Fractions

Review the examples in Exercise No. 277 on page 103 and No. 281 on page 104. Also perform the following subtractions.

108 THE ART OF CALCULATION

- | | | | |
|---------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 1. $\frac{2}{3} - \frac{1}{2}$ | 4. $1\frac{1}{2} - \frac{1}{4}$ | 7. $\frac{7}{24} - \frac{1}{8}$ | 10. $1\frac{1}{2} - \frac{7}{8}$ |
| 2. $1\frac{1}{2} - \frac{1}{2}$ | 5. $\frac{11}{12} - \frac{2}{4}$ | 8. $\frac{13}{24} - \frac{3}{8}$ | |
| 3. $\frac{5}{12} - \frac{1}{4}$ | 6. $1\frac{7}{8} - \frac{3}{4}$ | 9. $\frac{19}{24} - \frac{5}{8}$ | |

Exercise No. 290

Mental Division

- | | | |
|--------------------|---------------------|---------------------|
| 1. $1479 \div 721$ | 8. $1523 \div 451$ | 15. $3012 \div 685$ |
| 2. $2435 \div 832$ | 9. $1966 \div 562$ | 16. $3347 \div 656$ |
| 3. $2036 \div 943$ | 10. $2421 \div 673$ | 17. $4498 \div 761$ |
| 4. $387 \div 151$ | 11. $1156 \div 241$ | 18. $4924 \div 872$ |
| 5. $623 \div 262$ | 12. $1643 \div 352$ | 19. $5547 \div 983$ |
| 6. $745 \div 233$ | 13. $2128 \div 463$ | 20. $1067 \div 194$ |
| 7. $1134 \div 344$ | 14. $2581 \div 574$ | |

Exercise No. 291

Addition of Fractions

Review the examples in Exercise No. 279 on page 103, No. 283 on page 104 and No. 287 on page 107. Also perform the following additions.

- | | | | |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1. $\frac{2}{5} + \frac{1}{10}$ | 4. $\frac{2}{3} + \frac{2}{10}$ | 7. $\frac{4}{5} + \frac{7}{10}$ | 10. $\frac{1}{2} + \frac{2}{5}$ |
| 2. $\frac{2}{5} + \frac{2}{10}$ | 5. $\frac{4}{5} + \frac{1}{10}$ | 8. $\frac{4}{5} + \frac{2}{10}$ | |
| 3. $\frac{2}{5} + \frac{7}{10}$ | 6. $\frac{4}{5} + \frac{3}{10}$ | 9. $\frac{1}{2} + \frac{1}{5}$ | |

Exercise No. 292

Mental Multiplication

Multiply mentally the following.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 43×44 | 8. 43×52 | 15. 43×51 |
| 2. 53×45 | 9. 53×44 | 16. 53×52 |
| 3. 63×46 | 10. 63×45 | 17. 63×44 |
| 4. 73×47 | 11. 73×46 | 18. 78×45 |
| 5. 83×48 | 12. 83×47 | 19. 83×46 |
| 6. 93×49 | 13. 93×48 | 20. 93×47 |
| 7. 33×51 | 14. 33×49 | |

Exercise No. 293

Subtraction of Fractions

Review the examples in Exercise No. 281 on page 104 and No. 289 on page 108. Also do the following.

- | | | | |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1. $\frac{3}{4} - \frac{1}{8}$ | 4. $1\frac{1}{2} - \frac{7}{8}$ | 7. $1\frac{1}{2} - \frac{1}{2}$ | 10. $\frac{2}{3} - \frac{1}{4}$ |
| 2. $1\frac{5}{8} - \frac{3}{8}$ | 5. $\frac{7}{12} - \frac{1}{2}$ | 8. $1\frac{5}{8} - \frac{1}{2}$ | |
| 3. $1\frac{1}{4} - \frac{5}{8}$ | 6. $\frac{1}{2} - \frac{1}{2}$ | 9. $\frac{1}{2} - \frac{1}{4}$ | |

Exercise No. 294

Mental Division

Divide mentally the following.

- | | | |
|--------------------|---------------------|---------------------|
| 1. $444 \div 131$ | 8. $4716 \div 963$ | 15. $3573 \div 693$ |
| 2. $795 \div 242$ | 9. $815 \div 174$ | 16. $971 \div 141$ |
| 3. $1154 \div 353$ | 10. $1348 \div 285$ | 17. $1712 \div 252$ |
| 4. $1424 \div 464$ | 11. $1421 \div 255$ | 18. $2255 \div 363$ |
| 5. $1767 \div 571$ | 12. $2118 \div 366$ | 19. $2955 \div 474$ |
| 6. $3186 \div 740$ | 13. $2676 \div 471$ | 20. $3820 \div 585$ |
| 7. $3493 \div 852$ | 14. $3375 \div 582$ | |

Exercise No. 295

Addition of Fractions

Review the examples in Exercise No. 279 on page 103, No. 283 on page 104 and No. 292 on page 108. Also perform the following additions.

- | | | | |
|---------------------------------|---------------------------------|--------------------------------|---------------------------------|
| 1. $\frac{1}{2} + \frac{3}{8}$ | 4. $\frac{1}{2} + \frac{3}{10}$ | 7. $\frac{1}{4} + \frac{1}{8}$ | 10. $\frac{1}{4} + \frac{1}{5}$ |
| 2. $\frac{1}{2} + \frac{1}{4}$ | 5. $\frac{1}{2} + \frac{7}{10}$ | 8. $\frac{1}{4} + \frac{3}{8}$ | |
| 3. $\frac{1}{2} + \frac{1}{10}$ | 6. $\frac{1}{2} + \frac{9}{10}$ | 9. $\frac{1}{4} + \frac{3}{8}$ | |

Exercise No. 299**Addition of Fractions**

Review the examples in Exercise No. 283 on page 104, No. 292 on page 108 and No. 295 on page 109. Also perform the following additions.

1. $\frac{1}{4} + \frac{1}{10}$

4. $\frac{1}{4} + \frac{9}{10}$

7. $\frac{3}{4} + \frac{3}{5}$

10. $\frac{3}{4} + \frac{3}{10}$

2. $\frac{1}{4} + \frac{3}{10}$

5. $\frac{3}{4} + \frac{1}{5}$

8. $\frac{3}{4} + \frac{4}{5}$

3. $\frac{1}{4} + \frac{7}{10}$

6. $\frac{3}{4} + \frac{2}{5}$

9. $\frac{3}{4} + \frac{1}{10}$

Exercise No. 300**Mental Multiplication**

Multiply mentally the following.

1. 45×62

8. 45×69

15. 45×68

2. 55×63

9. 55×62

16. 55×69

3. 65×64

10. 65×63

17. 65×62

4. 75×65

11. 75×64

18. 75×63

5. 85×66

12. 85×65

19. 85×64

6. 95×67

13. 95×66

20. 95×65

7. 35×68

14. 35×67

Exercise No. 301**Subtraction of Fractions**

Review the examples in Exercise No. 293 on page 109 and No. 297 on page 110. Also perform the following subtractions.

1. $1\frac{1}{24} - \frac{3}{8}$

4. $1\frac{7}{24} - \frac{3}{8}$

7. $1\frac{5}{24} - \frac{5}{8}$

10. $1\frac{7}{24} - \frac{7}{8}$

2. $\frac{19}{24} - \frac{3}{8}$

5. $\frac{17}{24} - \frac{5}{8}$

8. $1\frac{13}{24} - \frac{5}{8}$

3. $\frac{23}{24} - \frac{3}{8}$

6. $1\frac{11}{24} - \frac{5}{8}$

9. $\frac{23}{24} - \frac{7}{8}$

Exercise No. 302**Mental Division**

Divide mentally the following.

1. $1714 \div 284$

3. $2714 \div 446$

5. $4617 \div 661$

2. $2399 \div 395$

4. $3507 \div 557$

6. $5303 \div 686$

112 THE ART OF CALCULATION

- | | | |
|---------------------|---------------------|---------------------|
| 7. $5886 \div 797$ | 12. $6588 \div 747$ | 17. $2502 \div 263$ |
| 8. $6665 \div 838$ | 13. $7189 \div 858$ | 18. $3440 \div 374$ |
| 9. $7233 \div 941$ | 14. $8238 \div 969$ | 19. $4450 \div 485$ |
| 10. $1084 \div 152$ | 15. $1385 \div 171$ | 20. $5423 \div 596$ |
| 11. $5757 \div 696$ | 16. $1493 \div 152$ | |

Exercise No. 303

Addition of Fractions

Review the examples in Exercise No. 292 on page 108, No. 295 on page 109 and No. 299 on page 111. Also perform the following additions.

- | | | | |
|---------------------------------|--------------------------------|---------------------------------|----------------------------------|
| 1. $\frac{3}{4} + \frac{7}{10}$ | 4. $\frac{1}{8} + \frac{2}{3}$ | 7. $\frac{1}{8} + \frac{1}{10}$ | 10. $\frac{1}{8} + \frac{2}{10}$ |
| 2. $\frac{3}{4} + \frac{2}{10}$ | 5. $\frac{1}{8} + \frac{2}{3}$ | 8. $\frac{1}{8} + \frac{3}{10}$ | |
| 3. $\frac{1}{8} + \frac{1}{5}$ | 6. $\frac{1}{8} + \frac{4}{5}$ | 9. $\frac{1}{8} + \frac{7}{10}$ | |

Exercise No. 304

Mental Multiplication

Multiply mentally the following.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 46×71 | 8. 46×78 | 15. 46×77 |
| 2. 56×72 | 9. 56×71 | 16. 56×78 |
| 3. 66×73 | 10. 66×72 | 17. 66×71 |
| 4. 76×74 | 11. 76×73 | 18. 76×72 |
| 5. 86×75 | 12. 86×74 | 19. 86×73 |
| 6. 96×76 | 13. 96×75 | 20. 96×74 |
| 7. 36×77 | 14. 36×76 | |

Exercise No. 305

Subtraction of Fractions

Review the examples in Exercise No. 297 on page 110 and No. 301 on page 111. Also perform the following subtractions.

- | | | | |
|---------------------------------|----------------------------------|----------------------------------|-----------------------------------|
| 1. $1\frac{1}{2} - \frac{7}{8}$ | 4. $\frac{1}{2} - \frac{1}{5}$ | 7. $\frac{1}{2} - \frac{2}{3}$ | 10. $1\frac{2}{10} - \frac{2}{3}$ |
| 2. $1\frac{1}{2} - \frac{7}{8}$ | 5. $\frac{2}{10} - \frac{1}{5}$ | 8. $\frac{7}{10} - \frac{2}{3}$ | |
| 3. $\frac{2}{10} - \frac{1}{5}$ | 6. $1\frac{1}{10} - \frac{1}{5}$ | 9. $1\frac{1}{10} - \frac{2}{3}$ | |

Exercise No. 306**Mental Division**

Divide mentally the following.

- | | | |
|--------------------|---------------------|---------------------|
| 1. $5338 \div 772$ | 8. $3606 \div 485$ | 15. $5954 \div 666$ |
| 2. $5393 \div 883$ | 9. $4518 \div 596$ | 16. $5887 \div 647$ |
| 3. $6001 \div 994$ | 10. $4711 \div 637$ | 17. $7123 \div 758$ |
| 4. $908 \div 145$ | 11. $2284 \div 282$ | 18. $8221 \div 869$ |
| 5. $1576 \div 256$ | 12. $3183 \div 393$ | 19. $9257 \div 973$ |
| 6. $1859 \div 263$ | 13. $3956 \div 444$ | 20. $1721 \div 184$ |
| 7. $2736 \div 374$ | 14. $4795 \div 555$ | |

Exercise No. 307**Addition of Fractions**

Review the examples in Exercise No. 295 on page 109, No. 297 on page 110 and No. 303 on page 112. Also perform the following additions.

- | | | | |
|--------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1. $\frac{3}{8} + \frac{1}{3}$ | 4. $\frac{3}{8} + \frac{4}{5}$ | 7. $\frac{3}{8} + \frac{7}{16}$ | 10. $\frac{5}{8} + \frac{3}{5}$ |
| 2. $\frac{3}{8} + \frac{2}{3}$ | 5. $\frac{3}{8} + \frac{1}{16}$ | 8. $\frac{3}{8} + \frac{9}{16}$ | |
| 3. $\frac{3}{8} + \frac{2}{3}$ | 6. $\frac{2}{8} + \frac{3}{16}$ | 9. $\frac{5}{8} + \frac{1}{3}$ | |

Exercise No. 308**Mental Multiplication**

Perform mentally the following multiplications.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 47×79 | 8. 47×87 | 15. 47×86 |
| 2. 57×81 | 9. 57×79 | 16. 57×87 |
| 3. 67×82 | 10. 67×81 | 17. 67×79 |
| 4. 77×83 | 11. 77×82 | 18. 77×81 |
| 5. 87×84 | 12. 87×83 | 19. 87×82 |
| 6. 97×85 | 13. 97×84 | 20. 97×83 |
| 7. 37×86 | 14. 37×85 | |

114 THE ART OF CALCULATION

Exercise No. 309

Subtraction of Fractions

Review the examples in Exercise No. 301 on page 111 and No. 305 on page 112. Also perform the following subtractions.

- | | | | |
|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 1. $\frac{7}{10} - \frac{2}{3}$ | 4. $1\frac{1}{2} - \frac{2}{3}$ | 7. $1\frac{1}{2} - \frac{4}{5}$ | 10. $\frac{9}{10} - \frac{1}{2}$ |
| 2. $\frac{9}{10} - \frac{2}{3}$ | 5. $\frac{9}{10} - \frac{4}{5}$ | 8. $1\frac{7}{10} - \frac{4}{5}$ | |
| 3. $1\frac{3}{10} - \frac{2}{3}$ | 6. $1\frac{1}{10} - \frac{4}{5}$ | 9. $\frac{7}{10} - \frac{1}{2}$ | |

Exercise No. 310

Mental Division

Divide mentally the following.

- | | | |
|--------------------|---------------------|---------------------|
| 1. $5365 \div 748$ | 8. $8304 \div 999$ | 15. $6720 \div 679$ |
| 2. $6599 \div 851$ | 9. $6075 \div 741$ | 16. $7831 \div 784$ |
| 3. $7445 \div 962$ | 10. $5241 \div 652$ | 17. $8917 \div 895$ |
| 4. $1243 \div 173$ | 11. $2682 \div 295$ | 18. $9441 \div 946$ |
| 5. $2220 \div 284$ | 12. $3411 \div 346$ | 19. $1563 \div 157$ |
| 6. $6293 \div 777$ | 13. $4471 \div 457$ | 20. $2627 \div 268$ |
| 7. $7548 \div 888$ | 14. $5667 \div 568$ | |

Exercise No. 311

Addition of Fractions

Review the examples in Exercise No. 297 on page 110, No. 303 on page 112 and No. 307 on page 113. Also add the following.

- | | | | |
|---------------------------------|---------------------------------|--------------------------------|---------------------------------|
| 1. $\frac{5}{8} + \frac{2}{3}$ | 4. $\frac{5}{8} + \frac{3}{10}$ | 7. $\frac{7}{8} + \frac{1}{5}$ | 10. $\frac{7}{8} + \frac{4}{5}$ |
| 2. $\frac{5}{8} + \frac{4}{5}$ | 5. $\frac{5}{8} + \frac{7}{10}$ | 8. $\frac{7}{8} + \frac{2}{5}$ | |
| 3. $\frac{5}{8} + \frac{1}{10}$ | 6. $\frac{5}{8} + \frac{9}{10}$ | 9. $\frac{7}{8} + \frac{3}{5}$ | |

Exercise No. 312

Mental Multiplication

Multiply mentally the following.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 48×88 | 8. 48×96 | 15. 48×95 |
| 2. 58×89 | 9. 58×88 | 16. 58×96 |
| 3. 68×91 | 10. 68×89 | 17. 68×88 |
| 4. 78×92 | 11. 78×91 | 18. 78×89 |
| 5. 88×93 | 12. 88×92 | 19. 88×91 |
| 6. 98×94 | 13. 98×93 | 20. 98×92 |
| 7. 38×95 | 14. 38×94 | |

Exercise No. 313**Subtraction of Fractions**

Review the examples in Exercise No. 305 on page 112 and No. 309 on page 114. Also perform the following subtractions.

- | | | | |
|----------------------------------|---------------------------------|----------------------------------|-----------------------------------|
| 1. $1\frac{1}{10} - \frac{1}{2}$ | 4. $\frac{4}{5} - \frac{1}{2}$ | 7. $\frac{9}{20} - \frac{1}{4}$ | 10. $1\frac{1}{20} - \frac{1}{4}$ |
| 2. $1\frac{3}{10} - \frac{1}{2}$ | 5. $1\frac{1}{3} - \frac{1}{2}$ | 8. $\frac{13}{30} - \frac{1}{4}$ | |
| 3. $\frac{3}{5} - \frac{1}{2}$ | 6. $1\frac{2}{3} - \frac{1}{2}$ | 9. $\frac{17}{30} - \frac{1}{4}$ | |

Exercise No. 314**Addition of Fractions**

Review the examples in Exercise No. 303 on page 112, No. 307 on page 113 and No. 311 on page 114. Also perform the following additions.

- | | | | |
|---------------------------------|---------------------------------|---------------------------------|-----------------------------------|
| 1. $\frac{7}{8} + \frac{1}{10}$ | 4. $\frac{7}{8} + \frac{9}{10}$ | 7. $\frac{1}{3} + \frac{2}{3}$ | 10. $\frac{1}{2} + 1\frac{3}{10}$ |
| 2. $\frac{7}{8} + \frac{3}{10}$ | 5. $\frac{1}{3} + \frac{1}{5}$ | 8. $\frac{1}{3} + \frac{4}{5}$ | |
| 3. $\frac{7}{8} + \frac{7}{10}$ | 6. $\frac{1}{3} + \frac{2}{5}$ | 9. $\frac{1}{3} + \frac{1}{10}$ | |

Exercise No. 315**Mental Multiplication**

Multiply the following mentally.

- | | | |
|-------------------|--------------------|--------------------|
| 1. 49×95 | 8. 49×97 | 15. 49×99 |
| 2. 59×96 | 9. 59×98 | 16. 59×95 |
| 3. 69×97 | 10. 69×99 | 17. 69×96 |
| 4. 79×98 | 11. 79×95 | 18. 79×97 |
| 5. 89×99 | 12. 89×96 | 19. 89×98 |
| 6. 99×95 | 13. 99×97 | 20. 99×99 |
| 7. 39×96 | 14. 39×98 | |

Exercise No. 316**Subtraction of Fractions**

Review the examples in Exercise No. 309 on page 114 and No. 313 on page 115. Also perform the following subtractions.

- | | | | |
|----------------------------------|----------------------------------|-----------------------------------|-----------------------------------|
| 1. $\frac{7}{30} - \frac{1}{4}$ | 4. $1\frac{3}{20} - \frac{1}{4}$ | 7. $1\frac{7}{20} - \frac{3}{4}$ | 10. $1\frac{1}{20} - \frac{3}{4}$ |
| 2. $\frac{11}{30} - \frac{1}{4}$ | 5. $\frac{19}{20} - \frac{3}{4}$ | 8. $1\frac{11}{20} - \frac{3}{4}$ | |
| 3. $\frac{19}{30} - \frac{1}{4}$ | 6. $1\frac{9}{20} - \frac{3}{4}$ | 9. $\frac{17}{20} - \frac{3}{4}$ | |

Exercise No. 317**Addition of Fractions**

Review the examples in Exercise No. 307 on page 113, No. 311 on page 114 and No. 314 on page 115. Also perform the following additions.

- | | | | |
|---------------------------------|--------------------------------|---------------------------------|----------------------------------|
| 1. $\frac{1}{3} + \frac{7}{10}$ | 4. $\frac{2}{3} + \frac{2}{3}$ | 7. $\frac{2}{3} + \frac{1}{10}$ | 10. $\frac{2}{3} + \frac{9}{10}$ |
| 2. $\frac{1}{3} + \frac{9}{10}$ | 5. $\frac{2}{3} + \frac{2}{3}$ | 8. $\frac{2}{3} + \frac{3}{10}$ | |
| 3. $\frac{2}{3} + \frac{1}{5}$ | 6. $\frac{2}{3} + \frac{1}{5}$ | 9. $\frac{2}{3} + \frac{7}{10}$ | |

Exercise No. 318**Subtraction of Fractions**

Review the examples in Exercise No. 313 on page 115 and No. 316 on this page. Also perform the following subtractions.

- | | | | |
|-----------------------------------|----------------------------------|----------------------------------|-----------------------------------|
| 1. $1\frac{9}{20} - \frac{3}{4}$ | 4. $\frac{21}{40} - \frac{1}{8}$ | 7. $\frac{9}{40} - \frac{1}{8}$ | 10. $1\frac{1}{40} - \frac{1}{8}$ |
| 2. $1\frac{13}{20} - \frac{3}{4}$ | 5. $\frac{29}{40} - \frac{1}{8}$ | 8. $\frac{17}{40} - \frac{1}{8}$ | |
| 3. $\frac{13}{40} - \frac{1}{8}$ | 6. $\frac{27}{40} - \frac{1}{8}$ | 9. $\frac{23}{40} - \frac{1}{8}$ | |

Exercise No. 319**Mental Division**

Divide the following mentally.

- | | | |
|-------------------|-------------------|-------------------|
| 1. $1066 \div 26$ | 3. $1708 \div 28$ | 5. $2511 \div 31$ |
| 2. $1377 \div 27$ | 4. $2059 \div 29$ | 6. $2912 \div 32$ |

- | | | |
|--------------------|--------------------|--------------------|
| 7. $1023 \div 33$ | 12. $2349 \div 29$ | 17. $1586 \div 26$ |
| 8. $1394 \div 34$ | 13. $2821 \div 31$ | 18. $1917 \div 27$ |
| 9. $1326 \div 26$ | 14. $992 \div 32$ | 19. $2268 \div 28$ |
| 10. $1647 \div 27$ | 15. $1353 \div 33$ | 20. $2639 \div 29$ |
| 11. $1988 \div 28$ | 16. $1734 \div 34$ | |

Exercise No. 320**Addition of Fractions**

Review the examples in Exercise No. 311 on page 114, No. 314 on page 115 and No. 315 on page 115. Also perform the following additions.

- | | | | |
|--------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1. $\frac{1}{6} + \frac{1}{3}$ | 4. $\frac{1}{6} + \frac{4}{3}$ | 7. $\frac{1}{6} + \frac{7}{10}$ | 10. $\frac{5}{8} + \frac{3}{2}$ |
| 2. $\frac{1}{6} + \frac{2}{3}$ | 5. $\frac{1}{6} + \frac{1}{10}$ | 8. $\frac{1}{6} + \frac{9}{10}$ | |
| 3. $\frac{1}{6} + \frac{3}{2}$ | 6. $\frac{1}{6} + \frac{3}{10}$ | 9. $\frac{5}{8} + \frac{1}{2}$ | |

Exercise No. 321**Subtraction of Fractions**

Review the examples in Exercise No. 314 on page 115, No. 316 on page 116 and No. 320 above. Also perform the following subtractions.

- | | | | |
|----------------------------------|----------------------------------|-----------------------------------|-----------------------------------|
| 1. $\frac{23}{40} - \frac{3}{8}$ | 4. $1\frac{7}{40} - \frac{3}{8}$ | 7. $1\frac{3}{40} - \frac{3}{8}$ | 10. $1\frac{1}{40} - \frac{5}{8}$ |
| 2. $\frac{31}{40} - \frac{3}{8}$ | 5. $\frac{19}{40} - \frac{3}{8}$ | 8. $1\frac{11}{40} - \frac{3}{8}$ | |
| 3. $\frac{39}{40} - \frac{3}{8}$ | 6. $\frac{27}{40} - \frac{3}{8}$ | 9. $\frac{33}{40} - \frac{5}{8}$ | |

Exercise No. 322**Mental Division**

Divide the following mentally.

- | | | |
|-------------------|--------------------|--------------------|
| 1. $1470 \div 35$ | 8. $1806 \div 43$ | 15. $1764 \div 42$ |
| 2. $1872 \div 36$ | 9. $1820 \div 35$ | 16. $2236 \div 43$ |
| 3. $2294 \div 37$ | 10. $2232 \div 36$ | 17. $2108 \div 34$ |
| 4. $2736 \div 38$ | 11. $2664 \div 37$ | 18. $2520 \div 35$ |
| 5. $3198 \div 39$ | 12. $3116 \div 38$ | 19. $2952 \div 36$ |
| 6. $3772 \div 41$ | 13. $3588 \div 39$ | 20. $3404 \div 37$ |
| 7. $1344 \div 42$ | 14. $1312 \div 41$ | |

Exercise No. 323**Addition of Fractions**

Review the examples in Exercise No. 314 on page 115, No. 317 on page 116 and No. 320 on page 117. Also perform the following additions.

1. $\frac{5}{8} + \frac{3}{8}$

3. $\frac{5}{8} + \frac{1}{10}$

5. $\frac{5}{8} + \frac{7}{16}$

2. $\frac{5}{8} + \frac{4}{8}$

4. $\frac{5}{8} + \frac{3}{10}$

6. $\frac{5}{8} + \frac{9}{16}$

Exercise No. 324**Subtraction of Fractions**

Review the examples in Exercise No. 318 on page 116 and No. 321 on page 117. Also perform the following subtractions.

1. $1\frac{9}{10} - \frac{5}{8}$

4. $\frac{37}{40} - \frac{5}{8}$

7. $1\frac{3}{40} - \frac{7}{8}$

10. $1\frac{27}{40} - \frac{7}{8}$

2. $1\frac{17}{16} - \frac{5}{8}$

5. $1\frac{11}{40} - \frac{5}{8}$

8. $1\frac{11}{40} - \frac{7}{8}$

3. $\frac{29}{16} - \frac{5}{8}$

6. $1\frac{21}{40} - \frac{5}{8}$

9. $1\frac{19}{40} - \frac{7}{8}$

Exercise No. 325**Mental Division**

Divide the following mentally.

1. $1892 \div 44$

8. $2236 \div 52$

15. $2193 \div 51$

2. $2385 \div 45$

9. $2332 \div 44$

16. $2756 \div 52$

3. $2898 \div 46$

10. $2835 \div 45$

17. $2772 \div 44$

4. $3431 \div 47$

11. $3358 \div 46$

18. $3285 \div 45$

5. $3984 \div 48$

12. $3901 \div 47$

19. $3818 \div 46$

6. $4557 \div 49$

13. $4464 \div 48$

20. $4371 \div 47$

7. $1683 \div 51$

14. $1617 \div 49$

Exercise No. 326**Addition of Fractions**

Review the examples in Exercise No. 317 on page 116, No. 320 on page 117 and No. 323 on this page.

Exercise No. 327**Subtraction of Fractions**

Review the examples in Exercise No. 321 on page 117 and No. 324 on page 118. Also perform the following subtractions.

- | | | | |
|-----------------------------------|-----------------------------------|----------------------------------|-----------------------------------|
| 1. $\frac{39}{40} - \frac{7}{8}$ | 4. $1\frac{31}{40} - \frac{7}{8}$ | 7. $\frac{14}{15} - \frac{1}{3}$ | 10. $\frac{19}{30} - \frac{1}{3}$ |
| 2. $1\frac{7}{40} - \frac{7}{8}$ | 5. $\frac{8}{15} - \frac{1}{3}$ | 8. $1\frac{2}{15} - \frac{1}{3}$ | |
| 3. $1\frac{23}{40} - \frac{7}{8}$ | 6. $\frac{11}{15} - \frac{1}{3}$ | 9. $\frac{13}{30} - \frac{1}{3}$ | |

Exercise No. 328**Mental Division**

Divide the following mentally.

- | | | |
|-------------------|--------------------|--------------------|
| 1. $2332 \div 53$ | 8. $2684 \div 61$ | 15. $2596 \div 59$ |
| 2. $2916 \div 54$ | 9. $2862 \div 53$ | 16. $3294 \div 61$ |
| 3. $3520 \div 55$ | 10. $3456 \div 54$ | 17. $3392 \div 53$ |
| 4. $4144 \div 56$ | 11. $4070 \div 55$ | 18. $3996 \div 54$ |
| 5. $4788 \div 57$ | 12. $4704 \div 56$ | 19. $4620 \div 55$ |
| 6. $5452 \div 58$ | 13. $5358 \div 57$ | 20. $5264 \div 56$ |
| 7. $2006 \div 59$ | 14. $1972 \div 58$ | |

Exercise No. 329**Addition of Fractions**

Review the examples in Exercise No. 320 on page 117 and 323 on page 118.

Exercise No. 330**Subtraction of Fractions**

Review the examples in Exercise No. 321 on page 117 and No. 324 on page 118. Also perform the following subtractions.

- | | | | |
|----------------------------------|----------------------------------|-----------------------------------|-----------------------------------|
| 1. $1\frac{1}{30} - \frac{1}{3}$ | 4. $1\frac{1}{15} - \frac{2}{3}$ | 7. $\frac{23}{30} - \frac{2}{3}$ | 10. $1\frac{7}{30} - \frac{2}{3}$ |
| 2. $1\frac{7}{30} - \frac{1}{3}$ | 5. $1\frac{4}{15} - \frac{2}{3}$ | 8. $\frac{29}{30} - \frac{2}{3}$ | |
| 3. $\frac{13}{15} - \frac{2}{3}$ | 6. $1\frac{7}{15} - \frac{2}{3}$ | 9. $1\frac{11}{30} - \frac{2}{3}$ | |

Exercise No. 331**Mental Division**

Divide the following mentally.

- | | | |
|-------------------|--------------------|--------------------|
| 1. $2790 \div 62$ | 8. $3105 \div 69$ | 15. $3060 \div 68$ |
| 2. $3465 \div 63$ | 9. $3410 \div 62$ | 16. $3795 \div 69$ |
| 3. $4160 \div 64$ | 10. $4095 \div 63$ | 17. $4030 \div 62$ |
| 4. $4875 \div 65$ | 11. $4800 \div 64$ | 18. $4725 \div 63$ |
| 5. $5610 \div 66$ | 12. $5525 \div 65$ | 19. $5440 \div 64$ |
| 6. $6365 \div 67$ | 13. $6270 \div 66$ | 20. $6175 \div 65$ |
| 7. $2380 \div 68$ | 14. $2345 \div 67$ | |

Exercise No. 332**Mental Division**

Divide the following mentally.

- | | | |
|-------------------|--------------------|--------------------|
| 1. $3266 \div 71$ | 8. $3588 \div 78$ | 15. $3542 \div 77$ |
| 2. $4032 \div 72$ | 9. $3976 \div 71$ | 16. $4368 \div 78$ |
| 3. $4818 \div 73$ | 10. $4752 \div 72$ | 17. $4686 \div 71$ |
| 4. $5624 \div 74$ | 11. $5548 \div 73$ | 18. $5472 \div 72$ |
| 5. $6450 \div 75$ | 12. $6364 \div 74$ | 19. $6278 \div 73$ |
| 6. $7296 \div 76$ | 13. $7200 \div 75$ | 20. $7104 \div 74$ |
| 7. $2772 \div 77$ | 14. $2736 \div 76$ | |

Exercise No. 333**Subtraction of Fractions**

Review the examples in Exercise No. 324 on page 118 and No. 330 on page 119. Also perform the following subtractions.

- | | | | |
|----------------------------------|----------------------------------|----------------------------------|-----------------------------------|
| 1. $\frac{11}{16} - \frac{1}{6}$ | 4. $\frac{29}{30} - \frac{1}{6}$ | 7. $\frac{13}{15} - \frac{1}{6}$ | 10. $1\frac{7}{30} - \frac{5}{8}$ |
| 2. $\frac{17}{18} - \frac{1}{6}$ | 5. $\frac{4}{15} - \frac{1}{6}$ | 8. $1\frac{1}{15} - \frac{1}{6}$ | |
| 3. $\frac{28}{30} - \frac{1}{6}$ | 6. $\frac{7}{15} - \frac{1}{6}$ | 9. $1\frac{1}{30} - \frac{5}{8}$ | |

Exercise No. 334**Mental Division**

Divide the following mentally.

- | | | |
|-------------------|-------------------|-------------------|
| 1. $3713 \div 79$ | 4. $6391 \div 83$ | 7. $3182 \div 86$ |
| 2. $4617 \div 81$ | 5. $7308 \div 84$ | 8. $4089 \div 87$ |
| 3. $5494 \div 82$ | 6. $8245 \div 85$ | 9. $4503 \div 79$ |

- | | | |
|--------------------|--------------------|--------------------|
| 10. $5427 \div 81$ | 14. $3145 \div 85$ | 18. $6237 \div 81$ |
| 11. $6314 \div 82$ | 15. $4042 \div 86$ | 19. $7134 \div 82$ |
| 12. $7221 \div 83$ | 16. $4959 \div 87$ | 20. $8051 \div 83$ |
| 13. $8148 \div 84$ | 17. $5293 \div 79$ | |

Exercise No. 335**Subtraction of Fractions**

Review the examples in Exercise No. 330 on page 119 and No. 333 on page 120. Also perform the following subtractions.

- | | | |
|---------------------------------|----------------------------------|----------------------------------|
| 1. $1\frac{3}{8} - \frac{5}{8}$ | 3. $1\frac{4}{5} - \frac{5}{8}$ | 5. $1\frac{8}{15} - \frac{5}{8}$ |
| 2. $1\frac{9}{8} - \frac{5}{8}$ | 4. $1\frac{2}{15} - \frac{5}{8}$ | 6. $1\frac{1}{15} - \frac{5}{8}$ |

Exercise No. 336**Mental Division**

Divide the following mentally.

- | | | |
|-------------------|--------------------|--------------------|
| 1. $4224 \div 88$ | 8. $4608 \div 96$ | 15. $4560 \div 95$ |
| 2. $5162 \div 89$ | 9. $5104 \div 88$ | 16. $5568 \div 96$ |
| 3. $6188 \div 91$ | 10. $6052 \div 89$ | 17. $5984 \div 88$ |
| 4. $7176 \div 92$ | 11. $7098 \div 91$ | 18. $6942 \div 89$ |
| 5. $8184 \div 93$ | 12. $8096 \div 92$ | 19. $8008 \div 91$ |
| 6. $9212 \div 94$ | 13. $9114 \div 93$ | 20. $9016 \div 92$ |
| 7. $3610 \div 95$ | 14. $3572 \div 94$ | |

Exercise No. 337**Mental Division**

Divide the following mentally.

- | | | |
|-------------------|--------------------|--------------------|
| 1. $4655 \div 95$ | 8. $4753 \div 97$ | 15. $4851 \div 99$ |
| 2. $5664 \div 96$ | 9. $5782 \div 98$ | 16. $5605 \div 95$ |
| 3. $6693 \div 97$ | 10. $6831 \div 99$ | 17. $6624 \div 96$ |
| 4. $7742 \div 98$ | 11. $7505 \div 95$ | 18. $7663 \div 97$ |
| 5. $8811 \div 99$ | 12. $8544 \div 96$ | 19. $8722 \div 98$ |
| 6. $9405 \div 95$ | 13. $9603 \div 97$ | 20. $9801 \div 99$ |
| 7. $3744 \div 96$ | 14. $3822 \div 98$ | |

DECIMALS IN GENERAL

For the purposes of this book our interest in decimals centers in the equivalence of value between certain decimals and common fractions. Decimal parts of a number that may be represented as simple fractions of that number are known as *aliquot parts* of it. Thus, $12\frac{1}{2}$, 25 and $33\frac{1}{3}$ are aliquot parts of 100, being respectively equal to $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{3}$ of 100.

A knowledge of aliquot parts simplifies many arithmetical calculations. Thus if it be required to multiply 7928 by 25, the simplest way is to annex two 0's to 7928, making it 792800, and then divide by 4, since 25 is $\frac{1}{4}$ of 100. The answer, which may easily be figured mentally, comes to 198200.

Again, if we wanted to know the cost of 25 gross of penholders at $66\frac{2}{3}\text{¢}$ per dozen, we would figure that 1 gross costs $\$2\frac{2}{3} \times 12$, or \$8, and that 25 gross therefore cost \$200.

Everybody with any degree of arithmetical training or experience is familiar with the equivalent decimal values for halves, quarters, eighths, thirds, sixths, fifths, tenths, twentieths, twenty-fifths and fiftieths. It is not difficult to extend the list of memorized values so as to include sixteenths and twelfths, and with this knowledge to make rapid calculations of values in thirty-seconds and twenty-fourths.

The succeeding exercises in decimals are designed toward this end. The student is drilled in representing the values of various fractions as decimals of an increasingly higher number of

places. No tables are given because values are more quickly learned by repeated calculation than by any effort at mere memorization.

Exercise No. 338

Two-Place Decimal Values

Express the following fractions as decimals of two places. Use fractional terminations where necessary. Thus, $\frac{1}{2}$ expressed as a two-place decimal becomes $.33\frac{1}{2}$.

- | | | | |
|------------------|------------------|------------------|-------------------|
| 1. $\frac{1}{8}$ | 4. $\frac{7}{8}$ | 7. $\frac{1}{6}$ | 10. $\frac{2}{3}$ |
| 2. $\frac{3}{8}$ | 5. $\frac{1}{3}$ | 8. $\frac{5}{6}$ | 11. $\frac{2}{3}$ |
| 3. $\frac{5}{8}$ | 6. $\frac{2}{3}$ | 9. $\frac{1}{5}$ | 12. $\frac{4}{5}$ |

Repeat this exercise three times.

Exercise No. 339

Multiplying Three Figures by Two

Multiply mentally the following.

No new principles are involved in multiplications of this type. The student is simply asked to apply the methods which he has already learned to larger numbers.

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. 111×26 | 4. 442×29 | 7. 721×33 | 10. 152×27 |
| 2. 222×27 | 5. 551×31 | 8. 832×34 | |
| 3. 331×28 | 6. 612×32 | 9. 941×26 | |

Exercise No. 340

Two-Place Decimal Values

Review the examples in Exercise No. 338 above.

Express the following as decimals of two places.

- | | | | |
|-------------------|--------------------|---------------------|--------------------|
| 1. $\frac{1}{16}$ | 5. $\frac{9}{16}$ | 9. $\frac{1}{12}$ | 13. $\frac{1}{8}$ |
| 2. $\frac{3}{16}$ | 6. $\frac{11}{16}$ | 10. $\frac{5}{12}$ | 14. $\frac{1}{24}$ |
| 3. $\frac{5}{16}$ | 7. $\frac{13}{16}$ | 11. $\frac{7}{12}$ | |
| 4. $\frac{7}{16}$ | 8. $\frac{15}{16}$ | 12. $\frac{11}{12}$ | |

Repeat this exercise three times.

124 THE ART OF CALCULATION

Exercise No. 341

Multiplying Three Figures by Two

Multiply mentally the following.

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. 121×35 | 4. 451×38 | 7. 731×42 | 10. 161×36 |
| 2. 232×36 | 5. 562×39 | 8. 842×43 | |
| 3. 343×37 | 6. 623×41 | 9. 953×35 | |

SHORT CUTS

There are a number of devices for shortening the work of calculation in specific cases, though most of the methods usually included under this head have only a limited practical value because they are applicable only in highly special cases. A few methods, like horizontal addition and combined addition and subtraction have first-class utility. A variety of short cuts of varying degrees of value are given in the following pages without any attempt to classify them. The student should become familiar with all of them because there is always benefit in viewing numbers from as many angles as possible.

Exercise No. 342

Horizontal Addition

The term *horizontal addition* is applied to the adding of numbers that are not arranged in column form. There is often an unnecessary waste of time in arranging numbers in the form of columns. This is particularly true when the numbers to be added are on bills, invoices, etc. Values on such papers may be totalled by writing down each partial sum as it is arrived at, and then making a final addition.

Consider the first of the following examples. The sum of the units is 37, the sum of the tens is 45, etc. The sums of the various orders are successively set down in the form shown below, and then added.

$$\begin{array}{r} 37 \\ 45 \\ 14 \\ 16 \\ \hline 17887 \end{array}$$

The process might of course be shortened somewhat by adding two orders at a time.

Add the following.

1. $\$32 + \$183 + \$54 + \$3486 + \$569 + \$9375 + \$85 + \4103
2. $\$875 + \$284 + \$37 + \$5200 + \$398 + \$62 + \$74 + \$2168 + \$720$
3. $763 + 827 + 49 + 5283 + 768 + 2175$
4. $1536 + 8973 + 5178 + 926 + 8259 + 36 + 867$
5. $9365 + 8375 + 1473 + 826 + 4123 + 15378$
6. $986 + 325 + 7261 + 5820 + 569 + 8371$
7. $6275 + 5183 + 985 + 3267 + 75 + 1528$
8. $1738 + 9168 + 8273 + 5298 + 9 + 6832 + 65$
9. $\$783.52 + \$41.27 + \$837.45 + \$9681.73 + \$48.26 + \$912.78 + \$91.75 + \$683.12 + \$41.83 + \$591.87 + \$291.83 + \$758.32 + \$58.67$
10. $46235 + 8976 + 5807 + 98397 + 68325 + 892 + 5140 + 6839 + 326 + 2125$

Exercise No. 343

Multiplying Three Figures by Two

Multiply mentally the following.

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. 131×44 | 4. 464×47 | 7. 743×51 | 10. 172×45 |
| 2. 242×45 | 5. 571×48 | 8. 854×52 | |
| 3. 353×46 | 6. 632×49 | 9. 961×44 | |

Exercise No. 344

Four-Place Decimal Values

Review the examples in Exercises No. 338 and 340 on page 123.

Express the fractions listed in Exercise No. 340 as decimals of four places. This is done by simply writing the value as parts of 100 of the terminal fractions of the proper two-place decimals. Thus, $\frac{1}{16}$, which is $.06\frac{1}{4}$ as a two-place decimal, becomes $.0625$ as a decimal of four places. Again, $\frac{1}{12}$ is $.08\frac{1}{3}$ or $.0833\frac{1}{3}$.

Exercise No. 345**Multiplying Three Figures by Two**

Multiply mentally the following.

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. 141×53 | 4. 474×56 | 7. 752×59 | 10. 185×54 |
| 2. 252×54 | 5. 585×57 | 8. 863×61 | |
| 3. 363×55 | 6. 641×58 | 9. 974×53 | |

Exercise No. 346**Combined Addition and Subtraction**

It sometimes becomes necessary to subtract the sum of several numbers from a single number. If the numbers to be added are arranged in column form, this may be done at what amounts to one operation by a very simple process.

The numbers may be arranged either as a sum with a missing addend, as in the examples given for practice, or else with the minuend written at the top with underscoring and the difference written at the bottom, as in the examples shown for illustration.

The so-called carry method of subtraction is used. The sum of each successive column is subtracted from the corresponding figure of the minuend plus as many tens as may be necessary to make the subtraction possible. The number of tens thus used is then added to the next column.

To illustrate: from 122808 take the sum of 35635, and 68921.

$$\begin{array}{r} 122808 \\ \hline \end{array}$$

$$\begin{array}{r} 35635 \\ \hline \end{array}$$

$$\begin{array}{r} 68921 \\ \hline \end{array}$$

$$\begin{array}{r} 18252 \end{array}$$

The sum of 5 and 1 is subtracted from 8; write 2 and carry 0. Subtract 5 from 10; write 5 and carry 1 because 1 ten was used to make the subtraction possible. With

128 THE ART OF CALCULATION

1 to carry, the next column adds to 16; subtract this from 18 and again carry 1. The next column adds to 14; subtract this from 22 and carry 2 because 2 tens were needed to make the subtraction possible in this case. Carrying 2 and subtracting from 12 gives the final necessary figure, 1.

The method of carrying may be made still more clear by taking an example that involves larger numbers; from 3744 subtract the sum of 366, 466, 566, 666, 766, 266 and 466.

3744

366

466

566

666

766

266

466

182

The sum of the first column, 42, is subtracted from 44 because 44 is the next higher number ending in 4 from which a subtraction can be made; 4 is carried. The sum of the second column, 46, is subtracted from 54 because 54 is the next higher number ending in 4 from which a subtraction can be made; 5 is carried. The sum of the hundreds' column subtracted from 39 leaves 1.

In the following examples fill in in each case the missing number that will make all the numbers add to the total shown.

1. \$24.96	2. 6016	3. \$29.44	4. 6144
6.24	376	7.36	384
1.56	141	1.84	24576
12.48	188	3.68	3072
.98	1504	58.88	145
3.12	752	1.38	49152
(?)	(?)	(?)	(?)
<u>\$149.18</u>	<u>105233</u>	<u>\$220.34</u>	<u>181777</u>

5. 864	6. \$168.86	7. \$475.17	8. \$286.09
108	10.56	46.82	5304.62
81	1.32	120.08	20463.20
5296	.96	2461.50	607.05
3456	2.64	500.07	6315.46
432	84.48	1208.92	73.90
(?)	(?)	(?)	(?)
<hr/> 11965	<hr/> \$944.66	<hr/> \$12933.16	<hr/> \$63452.87

Exercise No. 347

Multiplying Three Figures by Two

Multiply mentally the following.

- 151×62
- 262×63
- 373×64
- 484×65
- 595×66
- 656×67
- 761×68
- 872×69
- 983×62
- 194×63

Exercise No. 348

Five-Place Decimal Values

Review the examples in Exercises No. 338 and 340 on page 123 and No. 344 on page 126.

Express the following fractions as decimals of five places.

To find values in thirty-seconds, add $.0312\frac{1}{2}$ to the next lower value in sixteenths, etc. The calculation is clearer in the mind if both sixteenths and thirty-seconds are first thought of as decimals of four places. Changing the four-place answer to five places is the work of an instant.

To find values in twenty-fourths, add $.0416\frac{2}{3}$ to the next lower value in twelfths, etc. In writing answers, drop final $\frac{1}{3}$, and raise final $\frac{2}{3}$ to make the last figure a 7.

- $\frac{1}{32}$
- $\frac{3}{32}$
- $\frac{5}{32}$
- $\frac{7}{32}$
- $\frac{9}{32}$
- $\frac{11}{32}$
- $\frac{13}{32}$
- $\frac{15}{32}$
- $\frac{17}{32}$
- $\frac{19}{32}$
- $\frac{21}{32}$
- $\frac{23}{32}$
- $\frac{25}{32}$
- $\frac{27}{32}$
- $\frac{29}{32}$

130 THE ART OF CALCULATION

- | | | | | |
|---------------------|--------------------|---------------------|---------------------|---------------------|
| 16. $\frac{11}{31}$ | 18. $\frac{5}{31}$ | 20. $\frac{11}{31}$ | 22. $\frac{17}{31}$ | 24. $\frac{11}{31}$ |
| 17. $\frac{1}{31}$ | 19. $\frac{7}{31}$ | 21. $\frac{13}{31}$ | 23. $\frac{19}{31}$ | |

Exercise No. 349

Multiplying Three Figures by Two

Multiply mentally the following.

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. 141×71 | 4. 474×74 | 7. 747×77 | 10. 173×72 |
| 2. 252×72 | 5. 585×75 | 8. 851×78 | |
| 3. 363×73 | 6. 696×76 | 9. 962×71 | |

Exercise No. 350

Multiplying by a Near Number

It sometimes happens that a multiplier is a little more or a little less than 100, 1000, 10000, etc. In cases of this kind it is quickest to multiply by the round number and then add or subtract the necessary difference. For example, multiply \$385.20 by 998. We multiply the dollar value by 1000 and subtract from this product twice \$385.20, thus:

$$\begin{array}{r}
 \$385200 \\
 \underline{770.40} \\
 \$384429.60
 \end{array}$$

Multiply the following. The student should be able to do most of these mentally.

- | | | |
|--------------------------|---------------------------|-------------------------|
| 1. $\$425 \times 999$ | 4. $\$258.30 \times 104$ | 7. $\$989 \times 992$ |
| 2. $\$865 \times 98$ | 5. $\$827.58 \times 1003$ | 8. $\$99 \times 97$ |
| 3. $\$735.25 \times 998$ | 6. $\$516 \times 1.02$ | 9. $\$1005 \times 1002$ |

Exercise No. 351

Multiplying Three Figures by Two

Multiply mentally the following.

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. 131×79 | 4. 464×83 | 7. 797×86 | 10. 152×81 |
| 2. 242×81 | 5. 575×84 | 8. 838×87 | |
| 3. 353×82 | 6. 686×85 | 9. 941×79 | |

Exercise No. 352

Review of Decimals

Review the examples in Exercise No. 340 on page 123, No. 344 on page 126 and No. 348 on page 129.

Exercise No. 353

Multiplying Three Figures by Two

Multiply mentally the following.

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. 141×88 | 4. 474×92 | 7. 747×95 | 10. 171×89 |
| 2. 252×89 | 5. 585×93 | 8. 858×96 | |
| 3. 363×91 | 6. 696×94 | 9. 969×88 | |

Exercise No. 354

Aliquot Parts in Multiplication

Reference has already been made to the fact that multiplication may be simplified by considering one of the factors as an aliquot part of some number ending in two or more 0's. Thus, 628×25 would be solved by multiplying 628 by 100 and dividing by 4; the answer comes to 15700. Again, multiplying 56×75 would be done most quickly by taking $\frac{3}{4}$ of 56 and then multiplying by 100.

Perform the following multiplications by the method of aliquot parts.

- | | | |
|---------------------|----------------------|---------------------------------|
| 1. $\$35 \times 15$ | 6. $\$36 \times 25$ | 11. $\$35 \times 18$ |
| 2. $\$42 \times 18$ | 7. $\$52 \times 250$ | 12. $\$28 \times 450$ |
| 3. $\$24 \times 16$ | 8. $\$42 \times 350$ | 13. $\$36 \times 33\frac{1}{2}$ |
| 4. $\$18 \times 45$ | 9. $\$150 \times 48$ | 14. $\$72 \times 16\frac{2}{3}$ |
| 5. $\$72 \times 75$ | 10. $\$64 \times 25$ | 15. $\$96 \times 12\frac{1}{2}$ |

Exercise No. 355**Multiplying Three Figures by Two**

Multiply mentally the following. Do not use short cuts.

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. 152×95 | 4. 485×98 | 7. 758×96 | 10. 194×99 |
| 2. 263×96 | 5. 596×99 | 8. 869×97 | |
| 3. 374×97 | 6. 647×95 | 9. 973×98 | |

Exercise No. 356**Review of Decimals**

Review the examples in Exercise No. 344 on page 126 and No. 348 on page 129.

Exercise No. 357**Multiplying Three Figures by Three**

Multiply mentally the following. Add together the first two partial products before determining the third.

- | | | |
|---------------------|---------------------|----------------------|
| 1. 111×101 | 5. 551×141 | 9. 941×181 |
| 2. 222×111 | 6. 612×151 | 10. 152×191 |
| 3. 331×121 | 7. 721×161 | |
| 4. 442×131 | 8. 832×171 | |

Exercise No. 358**Simplifying the Multiplier**

Sometimes a multiplier is of such a nature that one part of it may be taken as an exact multiple of another. In such cases an operation is eliminated by making a single multiplication of the first-found partial product instead of two multiplications of the original multiplicand. In the example at the left above, the 18 in the multiplier is equal to 3 times the 6. We therefore multiply the first partial product by 3 instead of multiplying the original multiplicand by 18. In the example at the right, 56 being equal

to 8 times 7, we multiply first by 8, placing the result in the proper position, and then multiply this partial product by 7.

2574	5462
<u>186</u>	<u>856</u>
15444	43696
<u>46332</u>	<u>305872</u>
478764	4675472

Multiply the following by this method.

- | | |
|---------------------------|-----------------------------|
| 1. $\$385.85 \times 642$ | 5. $\$9541.12 \times 546$ |
| 2. $\$742.50 \times 328$ | 6. $\$172.48 \times 763$ |
| 3. $\$82615 \times 729$ | 7. $\$2153.28 \times 18624$ |
| 4. $\$4265.25 \times 255$ | 8. $\$530.75 \times 16412$ |

Exercise No. 359

Multiplying Three Figures by Three

Multiply mentally the following.

- | | | |
|---------------------|---------------------|----------------------|
| 1. 121×202 | 5. 562×242 | 9. 953×282 |
| 2. 232×212 | 6. 623×252 | 10. 161×292 |
| 3. 343×222 | 7. 731×262 | |
| 4. 451×232 | 8. 842×272 | |

Exercise No. 360

Review of Decimals

Review the examples in Exercise No. 348 on page 129.

Exercise No. 361

Multiplying Three Figures by Three

Multiply mentally the following.

- | | | |
|---------------------|---------------------|----------------------|
| 1. 131×303 | 5. 571×343 | 9. 961×383 |
| 2. 242×313 | 6. 632×353 | 10. 172×393 |
| 3. 353×323 | 7. 743×363 | |
| 4. 464×333 | 8. 854×373 | |

134 THE ART OF CALCULATION

Exercise No. 362

Multiplication by Factoring

When a multiplier can be taken as the product of two factors, it may be quicker to make separate multiplications by each of these factors than to proceed in the ordinary manner. Take the example 632×156 . In the illustrations below, the one at the left shows the ordinary method. At the right the multiplier is split up into the factors 13 and 12; the multiplicand is multiplied by 13 and the result is then multiplied by 12.

632	632
<u>156</u>	<u>13</u>
3792	8216
3160	<u>12</u>
<u>632</u>	98592
98592	

Multiply the following by this method.

- | | | |
|---------------------|---------------------|---------------------|
| 1. 759×182 | 4. 656×285 | 7. 542×221 |
| 2. 684×169 | 5. 309×289 | 8. 327×224 |
| 3. 327×228 | 6. 728×324 | 9. 986×196 |

Exercise No. 363

Multiplying Three Figures by Three

Multiply mentally the following.

- | | | |
|---------------------|---------------------|----------------------|
| 1. 141×404 | 5. 585×444 | 9. 974×484 |
| 2. 252×414 | 6. 641×454 | 10. 185×494 |
| 3. 363×424 | 7. 752×464 | |
| 4. 474×434 | 8. 863×474 | |

Exercise No. 364

Factors Between 11 and 19

A quick way to calculate the product of two numbers between 11 and 19 is to add the units of one number to the whole of the other, annex 0 and add the product of the units of both numbers. Thus, to multiply 16×18 :

16 and 8 are 24; call this 240 and add 48, making 288. The same result would be reached by adding 6 to 18.

Multiply by this method:

- | | | |
|-------------------|-------------------|-------------------|
| 1. 14×15 | 4. 15×16 | 7. 16×17 |
| 2. 18×19 | 5. 13×15 | 8. 14×16 |
| 3. 15×17 | 6. 13×19 | 9. 19×19 |

Exercise No. 365

Multiplying Three Figures by Three

Multiply mentally the following.

- | | | |
|---------------------|---------------------|----------------------|
| 1. 151×505 | 5. 595×545 | 9. 983×585 |
| 2. 262×515 | 6. 656×555 | 10. 194×595 |
| 3. 373×525 | 7. 761×565 | |
| 4. 484×535 | 8. 872×575 | |

Exercise No. 366

Multiplying by 11

When the multiplicand consists of two figures the sum of which is less than 10, the product is found by writing the two figures of the multiplicand with their sum between them. Thus, to multiply 62 by 11 we write 6 and 2 with the sum of 6 and 2 between these figures, obtaining 682.

To multiply larger numbers by 11, apply the following rule. Beginning at the right, write the units' figure of the multiplicand, then successively the units plus the tens, the tens plus the hundreds, the hundreds plus the thousands, etc., carrying wherever necessary, and ending with the highest order of the multiplicand, or the highest order plus the carrying figure. Thus, to multiply 4762 by 11: write 2; add 2 and 6 and write 8; add 6 and 7, write 3 and carry 1; add 7 and 4, increase it by the 1 carried, write 2 and carry 1; add this 1 to 4 and write 5. Answer, 52382.

Multiply the following by this method.

- | | |
|---------------------------|---------------------------|
| 1. \$5136 \times 11 | 5. \$41268.45 \times 11 |
| 2. \$72638 \times 11 | 6. \$3275.75 \times 11 |
| 3. \$514832 \times 11 | 7. \$48263.25 \times 11 |
| 4. \$37281.05 \times 11 | 8. \$94873.30 \times 11 |

Exercise No. 367**Multiplying Three Figures by Three**

Multiply mentally the following.

- | | | |
|---------------------|---------------------|----------------------|
| 1. 141 \times 606 | 5. 585 \times 646 | 9. 962 \times 686 |
| 2. 252 \times 616 | 6. 696 \times 656 | 10. 173 \times 696 |
| 3. 363 \times 626 | 7. 747 \times 666 | |
| 4. 474 \times 636 | 8. 851 \times 676 | |

Exercise No. 368**Multiplying by 21, 31, 41, etc.**

Setting down the product from right to left, write the units' figure of the multiplicand, then multiply each order of the multiplicand by the tens' figure of the multiplier, increasing the result in each case by the next higher order of the multiplicand and any necessary carrying figure.

Example, multiply 387 by 41; write 7; multiply 7 by 4, add the 8 of the multiplicand, making 36, write 6 and carry 3; multiply 8 by 4, add the 3 of the multiplicand and the carried 3, making 38, write 8 and carry 3; multiply 3 by 4 and add the carried 3 making 15, write 15. Answer, 15867.

Multiply by this method:

- | | |
|--------------------------|--------------------------|
| 1. \$2735.50 \times 51 | 5. \$7415.40 \times 61 |
| 2. \$1824.75 \times 81 | 6. \$8291.25 \times 91 |
| 3. \$5104.30 \times 31 | 7. \$2134.15 \times 71 |
| 4. \$6238.65 \times 21 | 8. \$5827.80 \times 41 |

Exercise No. 369**Multiplying Three Figures by Three**

Multiply mentally the following.

- | | | |
|---------------------|---------------------|----------------------|
| 1. 131×707 | 5. 575×747 | 9. 941×787 |
| 2. 242×717 | 6. 686×757 | 10. 152×797 |
| 3. 353×727 | 7. 797×767 | |
| 4. 464×737 | 8. 838×777 | |

Exercise No. 370**Squares of Numbers**

The square of a number is the number multiplied by itself. Squares may be determined quickly if the given number is considered to be the sum of two numbers. In algebra such a sum would ordinarily be taken as $a + b$ and its square would be $a^2 + 2ab + b^2$. In regular arithmetical cases a becomes the tens of the number and b the units. Thus, 25 is $20 + 5$, and 146 is $140 + 6$. The algebraic formula for the square of the sum of two numbers is expressed as the square of the first plus twice the product of the first by the second plus the square of the second. Thus, 25 squared is 20×20 (400) plus $2 \times 20 \times 5$ (200) plus 5×5 (25); the total is 625.

In computing squares by this principle you may immediately annex the square of the second to the square of the first, and then add twice the product of the first by the second. Thus in squaring 25 you would immediately say 425, and then add to this $2 \times 20 \times 5$ (200), making 625. In squaring 146 you immediately say 19636 and add to this $2 \times 140 \times 6$ (1680), making 21316. Always allow two places for the square of the second. Thus in squaring 61 the first partial product is 3601, to which 120 is added to make 3721.

In squaring numbers on paper the following method will be found rapid where large numbers are involved. Set the given number down twice as if for regular multiplication. Assuming that it is considered to consist of tens and units,

138 THE ART OF CALCULATION

multiply units by units, write units in the result and carry the tens. Add the two given tens together, multiply this sum by the given units, add the carried figure, write tens in the result and carry hundreds. Multiply tens by tens, add the carried figure and write the result.

$$\begin{array}{r} 67 \\ 67 \\ \hline 4489 \end{array}$$

$$\begin{array}{r} 134 \\ 134 \\ \hline 17956 \end{array}$$

$$\begin{array}{r} 1613 \\ 1613 \\ \hline 2601769 \end{array}$$

In the first illustrative example at the left, $7 \times 7 = 49$, write 9 and carry 4; $6 + 6 = 12$, $12 \times 7 = 84$, $84 + 4 = 88$, write 8 and carry 8; $6 \times 6 = 36$, $36 + 8 = 44$.

In the second example, $4 \times 4 = 16$, write 6 and carry 1; $13 + 13 = 26$, $26 \times 4 = 104$, $104 + 1 = 105$, write 5 and carry 10; $13 \times 13 = 169$, $169 + 10 = 179$, write 179.

The third example is worked somewhat differently because here the parts of the number are considered to be 1600 and 13. $13 \times 13 = 169$, write 69 (two figures) and carry 1; $16 + 16 = 32$, $32 \times 13 = 416$, $416 + 1 = 417$, write 17 and carry 4; $16 \times 16 = 256$, $256 + 4 = 260$, write 260.

Find the squares of the following numbers. Do all the examples first by the first method, then by the second method.

- | | | | | |
|-------|--------|--------|----------|----------|
| 1. 74 | 4. 64 | 7. 124 | 10. 197 | 13. 1314 |
| 2. 93 | 5. 38 | 8. 146 | 11. 1112 | 14. 1516 |
| 3. 82 | 6. 112 | 9. 168 | 12. 1213 | 15. 1719 |

Exercise No. 371

Multiplying Three Figures by Three

Multiply mentally the following.

- | | | |
|---------------------|---------------------|----------------------|
| 1. 141×808 | 5. 585×848 | 9. 969×888 |
| 2. 252×818 | 6. 696×858 | 10. 171×898 |
| 3. 363×828 | 7. 747×868 | |
| 4. 474×838 | 8. 858×878 | |

Exercise No. 372

Multiplying When Units Are Alike

The following method is a variation of that explained in connection with the squaring of numbers.

$$\begin{array}{r} 47 \\ 67 \\ \hline 3149 \end{array} \qquad \begin{array}{r} 613 \\ 913 \\ \hline 559669 \end{array}$$

In the illustration at the left, $7 \times 7 = 49$, write 9 and carry 4; $6 + 4 = 10$, $10 \times 7 = 70$, $70 + 4 = 74$, write 4 and carry 7; $4 \times 6 = 24$, $24 + 7 = 31$, write 31.

In the illustration at the right, $13 \times 13 = 169$, write 69 and carry 1; $6 + 9 = 15$, $15 \times 13 = 195$, $195 + 1 = 196$, write 96 and carry 1; $6 \times 9 = 54$, $54 + 1 = 55$, write 55.

Perform the following multiplications by this method.

- | | | |
|--------------------|---------------------|---------------------|
| 1. 136×56 | 4. 195×115 | 7. 516×816 |
| 2. 159×79 | 5. 234×174 | 8. 714×314 |
| 3. 172×92 | 6. 217×197 | 9. 217×917 |

Exercise No. 373

Multiplying Three Figures by Three

- | | | |
|---------------------|---------------------|----------------------|
| 1. 152×909 | 5. 596×949 | 9. 973×989 |
| 2. 263×919 | 6. 647×959 | 10. 184×999 |
| 3. 374×929 | 7. 758×969 | |
| 4. 485×939 | 8. 869×979 | |

Exercise No. 374

Multiplying When Tens or Hundreds Are Alike

This is a variation of the method explained in Exercise No. 372 above.

$$\begin{array}{r} 83 \\ 89 \\ \hline 7387 \end{array} \qquad \begin{array}{r} 717 \\ 714 \\ \hline 511938 \end{array}$$

140 THE ART OF CALCULATION

In the example on page 139, $3 \times 9 = 27$, write 7 and carry 2; $3 + 9 = 12$, $12 \times 8 = 96$, $96 + 2 = 98$, write 8 and carry 9; $8 \times 8 = 64$, $64 + 9 = 73$, write 73.

In the example on page 139, $17 \times 14 = 238$, write 38 and carry 2; $17 + 14 = 31$, $31 \times 7 = 217$, $217 + 2 = 219$, write 19 and carry 2; $7 \times 7 = 49$, $49 + 2 = 51$, write 51.

Multiply the following by this method.

- | | | |
|-------------------|---------------------|---------------------|
| 1. 92×93 | 4. 92×97 | 7. 416×418 |
| 2. 62×65 | 5. 213×215 | 8. 509×519 |
| 3. 84×87 | 6. 321×312 | 9. 913×917 |

Exercise No. 375

Square of Numbers Ending in 5

If a number to be squared consists of tens and units, and if the units are 5, then twice the product of the first part by the second is equal to the given number of tens. Thus, in 25×25 , $20 \times 5 \times 2$ is equal to 20×10 ; in 35×35 , $30 \times 5 \times 2$ is equal to 30×10 . Accordingly when dealing with numbers of this type we may at once annex 25 to the product of the given tens multiplied by one more than the given tens. That is to say, $25 \times 25 = 625$, in which the 6 represents 3×2 ; $35 \times 35 = 1225$ in which the 12 represents 4×3 ; $45 \times 45 = 2025$, in which the 20 represents 5×4 , etc.

Find the squares of the following numbers by this method.

- | | | | | |
|-------|-------|--------|---------|---------|
| 1. 45 | 4. 75 | 7. 115 | 10. 175 | 13. 335 |
| 2. 55 | 5. 85 | 8. 135 | 11. 195 | 14. 355 |
| 3. 65 | 6. 95 | 9. 155 | 12. 315 | 15. 375 |

Exercise No. 376

Multiplying Like Tens with Units Making 10

The principle explained above applies to any case in which the tens are alike and the sum of the units is 10.

Thus the product of 46×44 is 2024. We arrive at this by multiplying 4×5 , making 20, and writing after this the product of 4×6 or 24.

Multiply in this manner the following.

- | | | |
|-------------------|---------------------|---------------------|
| 1. 23×27 | 4. 103×107 | 7. 178×172 |
| 2. 41×49 | 5. 112×118 | 8. 169×161 |
| 3. 36×34 | 6. 154×156 | 9. 192×198 |

Exercise No. 377

Squaring Numbers Ending in 25

When a number ends in 25, like 725 for instance, we may take it as the sum of two numbers of which one represents hundreds and the other tens and units. In such cases twice the product of the first part by the second is equal to 50 times the first part. The result of this multiplication is a certain number of thousands.

To find the square of 725 we first write 0625 after the square of 7, making 490625. To this we add as many thousands as are represented by 7×5 . $490625 + 35000 = 525625$.

Another method of finding these squares is by setting the numbers down as in the following illustration.

$$\begin{array}{r} 725 \\ 725 \\ \hline 525625 \end{array}$$

At once write 625 as the square of 25. Multiply 7 by 5, write 5 and carry 3; multiply 7 by 7, add 3, write 52.

Find the square of the following numbers by both of the foregoing methods.

- | | | | | |
|--------|---------|---------|---------|----------|
| 1. 525 | 3. 825 | 5. 1225 | 7. 1625 | 9. 1825 |
| 2. 625 | 4. 1025 | 6. 1325 | 8. 1725 | 10. 1925 |

Exercise No. 378

Multiplying a Sum by a Difference

The algebraic product of $a + b$ and $a - b$ is $a^2 - b^2$. When numbers to be multiplied can be expressed as the sum of and the difference between two numbers, the product equals the square of the first minus the square of the second. Thus 63×57 may be expressed as $60 + 3$ multiplied by $60 - 3$. The product equals 60×60 minus 3×3 . This comes to $3600 - 9$ or 3591.

There is no limit to the combinations of numbers for which this principle would hold true, but for practical purposes we may be satisfied to recognize those in which the units add to 10 and the tens have a difference of 1.

Multiply the following by this method.

- | | | |
|-------------------|---------------------|---------------------|
| 1. 72×68 | 4. 101×119 | 7. 152×168 |
| 2. 83×77 | 5. 123×137 | 8. 173×187 |
| 3. 94×86 | 6. 146×154 | 9. 182×198 |

Exercise No. 379

Multiplying Mixed Numbers with Like Integers

When integers are alike in mixed numbers, as in $9\frac{1}{4} \times 9\frac{3}{4}$, their product is found by multiplying one integer by the other plus the sum of the two fractions; to this partial product add that obtained by multiplying together the two fractions.

$$\begin{array}{r}
 9\frac{1}{4} \\
 9\frac{3}{4} \\
 \hline
 90\frac{3}{16}
 \end{array}
 \qquad
 \begin{array}{r}
 8\frac{3}{4} \\
 8\frac{5}{8} \\
 \hline
 76\frac{2}{3} \\
 \frac{5}{8} \\
 \hline
 77\frac{7}{24}
 \end{array}$$

In the illustrative example at the left, 9 is multiplied by $9 + \frac{1}{4} + \frac{3}{4}$, or 10. The product of this is 90, and to 90 is added the product of $\frac{1}{4}$ and $\frac{3}{4}$, or $\frac{3}{16}$.

In the second example 8 is multiplied by $8 + \frac{3}{4} + \frac{5}{8}$, or $9\frac{7}{8}$, producing $76\frac{2}{3}$. To this is added the product of $\frac{3}{4} \times \frac{5}{8}$, or $\frac{15}{32}$, making a total of $77\frac{7}{24}$.

Multiply the following.

- | | | |
|---|---|--|
| 1. $9\frac{1}{2} \times 9\frac{3}{4}$ | 5. $3\frac{1}{2} \times 3\frac{3}{4}$ | 9. $5\frac{1}{2} \times 5\frac{1}{2}$ |
| 2. $10\frac{2}{3} \times 10\frac{2}{3}$ | 6. $60\frac{2}{3} \times 60\frac{2}{3}$ | 10. $8\frac{1}{4} \times 8\frac{1}{4}$ |
| 3. $12\frac{5}{8} \times 12\frac{1}{2}$ | 7. $40\frac{3}{4} \times 40\frac{1}{4}$ | 11. $6\frac{1}{2} \times 6\frac{1}{2}$ |
| 4. $18\frac{1}{2} \times 18\frac{1}{2}$ | 8. $25\frac{2}{3} \times 25\frac{2}{3}$ | 12. $12\frac{1}{3} \times 12\frac{2}{3}$ |

Exercise No. 380

Multiplying by a Number Nearly Whole

Sometimes a multiplier lacks a single fractional unit of being a whole number. Examples would be $5\frac{2}{3}$, $6\frac{1}{4}$ and $7\frac{1}{2}$, which respectively lack $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{1}{2}$ of being 6, 7 and 8. In cases of this kind raise the multiplier to the next larger whole number, and after multiplying the multiplicand by this number, subtract from the product the necessary fractional part of the multiplicand. Thus, to multiply 64 by $3\frac{1}{4}$, we multiply 64 by 4, obtaining 256, and from this we subtract $\frac{1}{4}$ of 64, or 16, arriving at a final result of 240.

Multiply by this method the following.

- | | | |
|------------------------------|------------------------------|------------------------------|
| 1. $48 \times 5\frac{1}{2}$ | 4. $250 \times 3\frac{1}{2}$ | 7. $180 \times 7\frac{1}{2}$ |
| 2. $75 \times 10\frac{2}{3}$ | 5. $522 \times 4\frac{3}{4}$ | 8. $720 \times 2\frac{1}{4}$ |
| 3. $136 \times 6\frac{2}{3}$ | 6. $672 \times 8\frac{1}{2}$ | 9. $342 \times 9\frac{1}{2}$ |

Exercise No. 381

Aliquot Parts in Division

The method of aliquot parts is as applicable to division as it is to multiplication. In ordinary cases we determine how many times the given divisor is contained exactly in some multiple of 10. We multiply the given dividend by the result of such division, and point off the product decimally in such a way as to express division by the proper multiple of 10. Thus, to divide 1840 by 25, we obtain a multiplier of 4 by dividing 25 into 100. Multiplying 1840 by 4 we get 7360, and dividing this decimally by 100 we obtain 73.60

144 THE ART OF CALCULATION

$$6375 \div 7\frac{1}{2}$$

$$\begin{array}{r} 6375 \\ 2125 \\ \hline 850.0 \end{array}$$

Another method of using aliquot parts is illustrated by the example shown above. The problem is to divide 6375 by $7\frac{1}{2}$. We note that $7\frac{1}{2}$ lacks one-third of itself of being 10. We therefore add one-third of itself to 6375 and divide the resulting sum decimally by 10.

Divide by the foregoing methods:

- | | | |
|-----------------------------|-----------------------------|----------------------------|
| 1. $580 \div 25$ | 4. $875 \div 250$ | 7. $1527 \div 150$ |
| 2. $750 \div 16\frac{2}{3}$ | 5. $640 \div 125$ | 8. $918 \div 15$ |
| 3. $450 \div 12\frac{1}{2}$ | 6. $435 \div 33\frac{1}{3}$ | 9. $582 \div 7\frac{1}{2}$ |

Exercise No. 382

Cubes of Numbers

The algebraic formula for the cube of the sum of two numbers, a and b , is $a^3 + 3a^2b + 3ab^2 + b^3$. This may be expressed as the cube of the first plus three times the square of the first multiplied by the second, plus three times the first multiplied by the square of the second plus the cube of the second.

By applying this formula it is not difficult to calculate mentally the cubes of numbers of two places. Suppose, for instance, that we want to find the cube of 26. We immediately annex the cube of 6 (216) to the cube of 2 (8), obtaining 8216. (Always allow three places for the cube of the second.) Multiplying 3×400 (square of 20) $\times 6$, we get 7200, which, added to 8216, makes 15416. Multiplying $3 \times 20 \times 36$ (square of 6) we obtain 2160, which, added to 15416 gives 17576 as the cube of 26.

Cubes may be readily written down from right to left by using a different method.

26^3	$6 \times 6 \times 6 = 216$	6
17576	$(6 \times 6 \times 2 \times 3) + 21 = 237$	7
	$(6 \times 2 \times 2 \times 3) + 23 = 95$	5
	$(2 \times 2 \times 2) + 9 = 17$	17

All the necessary writing is shown on p.144 at the left. The method of making the calculation is analyzed at the right. The cube of 6 is 216, write 6 and carry 21. The square of 6 (36) multiplied by 2 (72) multiplied by 3 (216) plus 21 comes to 237, write 7 and carry 23. The product of 6 times the square of 2 (24) multiplied by 3 (72) plus 23 comes to 95, write 5 and carry 9. The cube of 2 is 8, which, added to 9, makes 17.

Before attempting the examples which follow the student ought to make himself thoroughly familiar with the cubes of the numbers from 1 to 9, so that he will not have to slow up to make such computations in the course of the example.

Find the cubes of the following numbers by both of the foregoing methods.

1. 14	4. 46	7. 65	10. 84	13. 95
2. 27	5. 59	8. 71	11. 86	14. 97
3. 33	6. 62	9. 73	12. 88	15. 99

Exercise No. 383

Algebraic Multiplication

Arithmetical products may be directly written down from right to left by using the method of cross-multiplication employed in algebra. A certain pattern is followed in multiplying each figure by every other figure. The operations are best explained by illustration.

	47	345
	26	678
<hr/>	<hr/>	<hr/>
	1222	234910

In the example at the left, $7 \times 6 = 42$, write 2 and carry 4; 4 plus 4×6 (28) plus 2×7 comes to 42, write 2 and carry 4; 4 plus 4×2 is 12, write 12. (It is best to start each part of the calculation with the carried number, which otherwise might not be easy to remember.)

In the second example, multiply 5×8 ; then 4×8 and 7×5 ; then 3×8 , 6×5 and 4×7 ; then 3×7 and 6×4 ; finally 3×6 . Carry as may be necessary.

THE ART OF CALCULATION

Table IV
Prime and Composite Numbers

1 Prime	41 Prime	71 Prime	98 = 2 × 49
2 Prime	42 = 2 × 21	72 = 2 × 36	7 × 14
3 Prime	3 × 14	3 × 24	99 = 3 × 33
4 = 2 × 2	6 × 7	4 × 18	9 × 11
5 Prime	43 Prime	6 × 12	100 = 2 × 50
6 = 2 × 3	44 = 2 × 22	8 × 9	4 × 25
7 Prime	4 × 11	73 Prime	5 × 20
8 = 2 × 4	45 = 3 × 15	74 = 2 × 37	10 × 10
9 = 3 × 3	5 × 9	75 = 3 × 25	101 Prime
10 = 2 × 5	46 = 2 × 23	5 × 15	102 = 2 × 51
11 Prime	47 Prime	76 = 2 × 38	3 × 34
12 = 2 × 6	48 = 2 × 24	4 × 19	6 × 17
3 × 4	3 × 16	77 = 7 × 11	103 Prime
13 Prime	4 × 12	78 = 2 × 39	104 = 2 × 52
14 = 2 × 7	6 × 8	3 × 26	4 × 26
15 = 3 × 5	49 = 7 × 7	6 × 13	8 × 13
16 = 2 × 8	50 = 2 × 25	79 Prime	105 = 3 × 35
4 = 4	5 × 10	80 = 2 × 40	5 × 21
17 Prime	51 = 3 × 17	4 × 20	7 × 15
18 = 2 × 9	52 = 2 × 26	5 × 16	106 = 2 × 53
3 × 6	4 × 13	8 × 10	107 Prime
19 Prime	53 Prime	81 = 3 × 27	108 = 2 × 54
20 = 2 × 10	54 = 2 × 27	9 × 9	3 × 36
4 × 5	3 × 18	82 = 2 × 41	4 × 27
21 = 3 × 7	6 × 9	83 Prime	6 × 18
22 = 2 × 11	55 = 5 × 11	84 = 2 × 42	9 × 12
23 Prime	56 = 2 × 28	3 × 28	109 Prime
24 = 2 × 12	4 × 14	4 × 21	110 = 2 × 55
3 × 8	7 × 8	6 × 14	5 × 22
4 × 6	57 = 3 × 19	7 × 12	10 × 11
25 = 5 × 5	58 = 2 × 29	85 = 5 × 17	111 = 3 × 37
26 = 2 × 13	59 Prime	86 = 2 × 43	112 = 2 × 56
27 = 3 × 9	60 = 2 × 30	87 = 3 × 29	4 × 28
28 = 2 × 14	3 × 20	88 = 2 × 44	7 × 16
4 × 7	4 × 15	4 × 22	8 × 14
29 Prime	5 × 12	8 × 11	113 Prime
30 = 2 × 15	6 × 10	89 Prime	114 = 2 × 57
3 × 10	61 Prime	90 = 2 × 45	3 × 38
5 × 6	62 = 2 × 31	3 × 30	6 × 19
31 Prime	63 = 3 × 21	5 × 18	115 = 5 × 23
32 = 2 × 16	7 × 9	6 × 15	116 = 2 × 58
4 × 8	64 = 2 × 32	9 × 10	4 × 29
33 = 3 × 11	4 × 16	91 = 7 × 13	117 = 3 × 39
34 = 2 × 17	8 × 8	92 = 2 × 46	9 × 13
35 = 5 × 7	65 = 5 × 13	4 × 23	118 = 2 × 59
36 = 2 × 18	66 = 2 × 33	93 = 3 × 31	119 = 7 × 17
3 × 12	3 × 22	94 = 2 × 47	120 = 2 × 60
4 × 9	6 × 11	95 = 5 × 19	3 × 40
6 × 6	67 Prime	96 = 2 × 48	4 × 30
37 Prime	68 = 2 × 34	3 × 32	5 × 24
38 = 2 × 19	4 × 17	4 × 24	6 × 20
39 = 3 × 13	69 = 3 × 23	6 × 16	8 × 15
40 = 2 × 20	70 = 2 × 35	8 × 12	10 × 12
4 × 10	5 × 14	97 Prime	121 = 11 × 11
5 × 8	7 × 10		122 = 2 × 61

Table IV (Continued)

123 = 3 × 41	149 Prime	173 Prime	196 = 2 × 93
124 = 2 × 62	150 = 2 × 75	174 = 2 × 87	4 × 49
4 × 31	3 × 50	3 × 58	7 × 28
125 = 5 × 25	5 × 30	6 × 29	14 × 14
126 = 2 × 63	6 × 25	175 = 5 × 35	197 Prime
3 × 42	10 × 15	7 × 25	198 = 2 × 99
6 × 21	151 Prime	176 = 2 × 88	3 × 66
7 × 18	152 = 2 × 76	4 × 44	6 × 33
9 × 14	4 × 38	8 × 22	9 × 22
127 Prime	8 × 19	11 × 16	11 × 18
128 = 2 × 64	153 = 3 × 51	177 = 3 × 59	199 Prime
4 × 32	9 × 17	178 = 2 × 89	200 = 2 × 100
8 × 16	154 = 2 × 77	179 Prime	4 × 50
129 = 3 × 43	7 × 22	180 = 2 × 90	5 × 40
130 = 2 × 65	11 × 14	3 × 60	8 × 25
5 × 26	155 = 5 × 31	4 × 45	10 × 20
10 × 13	156 = 2 × 78	5 × 36	201 = 3 × 67
131 Prime	3 × 52	6 × 30	202 = 2 × 101
132 = 2 × 66	4 × 39	9 × 20	203 = 7 × 29
3 × 44	6 × 26	10 × 18	204 = 2 × 102
4 × 33	12 × 13	12 × 15	3 × 68
6 × 22	157 Prime	181 Prime	4 × 51
11 × 12	158 = 2 × 79	182 = 2 × 91	6 × 34
133 = 7 × 19	159 = 3 × 53	7 × 26	12 × 17
134 = 2 × 67	160 = 2 × 80	13 × 14	205 = 5 × 41
135 = 3 × 45	4 × 40	183 = 3 × 61	206 = 2 × 103
5 × 27	5 × 32	184 = 2 × 92	207 = 3 × 69
9 × 15	8 × 20	4 × 46	9 × 23
136 = 2 × 68	10 × 16	8 × 23	208 = 2 × 104
4 × 34	161 = 7 × 23	185 = 5 × 37	4 × 52
8 × 17	162 = 2 × 81	186 = 2 × 93	8 × 26
137 Prime	3 × 54	3 × 62	13 × 16
138 = 2 × 69	6 × 27	6 × 31	209 = 11 × 19
3 × 46	9 × 18	187 = 11 × 17	210 = 2 × 105
6 × 23	163 Prime	188 = 2 × 94	3 × 70
139 Prime	164 = 2 × 82	4 × 47	5 × 42
140 = 2 × 70	4 × 41	189 = 3 × 63	6 × 35
4 × 35	165 = 3 × 55	7 × 27	7 × 30
5 × 28	5 × 33	9 × 21	10 × 21
7 × 20	11 × 15	190 = 2 × 95	14 × 15
10 × 14	166 = 2 × 83	5 × 38	211 Prime
141 = 3 × 47	167 Prime	10 × 19	212 = 2 × 106
142 = 2 × 71	168 = 2 × 84	191 Prime	4 × 53
143 = 11 × 13	3 × 56	192 = 2 × 96	213 = 3 × 71
144 = 2 × 72	4 × 42	3 × 64	214 = 2 × 107
3 × 48	6 × 28	4 × 48	215 = 5 × 43
4 × 36	7 × 24	6 × 32	216 = 2 × 108
6 × 24	8 × 21	8 × 24	3 × 72
8 × 18	12 × 14	12 × 16	4 × 54
9 × 16	169 = 13 × 13	193 Prime	6 × 36
12 × 12	170 = 2 × 85	194 = 2 × 97	8 × 27
145 = 5 × 29	5 × 34	195 = 3 × 65	9 × 24
146 = 2 × 73	10 × 17	5 × 39	12 × 18
147 = 3 × 49	171 = 3 × 57	13 × 15	217 = 7 × 31
7 × 21	9 × 19		218 = 2 × 109
148 = 2 × 74	172 = 2 × 86		219 = 3 × 73
4 × 37	4 × 43		

Table IV (Continued)

220 = 2×110	240 = 2×120	261 = 3×87	283 Prime
4 \times 55	3 \times 80	9 \times 29	284 = 2×142
5 \times 44	4 \times 60	262 = 2×131	4 \times 71
10 \times 22	5 \times 48	263 Prime	285 = 3×95
11 \times 20	6 \times 40	264 = 2×132	5 \times 57
221 = 13×17	8 \times 30	3 \times 88	15 \times 19
222 = 2×111	10 \times 24	4 \times 66	286 = 2×143
3 \times 74	12 \times 20	6 \times 44	11 \times 26
6 \times 37	15 \times 16	8 \times 33	13 \times 22
223 Prime	241 Prime	11 \times 24	287 = 7×41
224 = 2×112	242 = 2×121	12 \times 22	288 = 2×144
4 \times 56	11 \times 22	265 = 5×53	3 \times 96
7 \times 32	243 = 3×81	266 = 2×133	4 \times 72
8 \times 28	9 \times 27	7 \times 38	6 \times 48
14 \times 16	244 = 2×122	14 \times 19	8 \times 36
225 = 3×75	4 \times 61	267 = 3×89	9 \times 32
5 \times 45	245 = 5×49	268 = 2×134	12 \times 24
9 \times 25	7 \times 35	4 \times 67	16 \times 18
15 \times 15	246 = 2×123	269 Prime	289 = 17×17
226 = 2×113	3 \times 82	270 = 2×135	290 = 2×145
227 Prime	6 \times 41	3 \times 90	5 \times 58
228 = 2×114	247 = 13×19	5 \times 54	10 \times 29
3 \times 76	248 = 2×124	6 \times 45	291 = 3×97
4 \times 57	4 \times 62	9 \times 30	292 = 2×146
6 \times 38	8 \times 31	10 \times 27	4 \times 73
12 \times 19	249 = 3×83	15 \times 18	293 Prime
229 Prime	250 = 2×125	271 Prime	294 = 2×147
230 = 2×115	5 \times 50	272 = 2×136	3 \times 98
5 \times 46	10 \times 25	4 \times 68	6 \times 49
10 \times 23	251 Prime	8 \times 34	7 \times 42
231 = 3×77	252 = 2×126	16 \times 17	14 \times 21
7 \times 33	3 \times 84	273 = 3×91	295 = 5×59
11 \times 21	4 \times 63	7 \times 39	296 = 2×148
232 = 2×116	6 \times 42	13 \times 21	4 \times 74
4 \times 58	7 \times 36	274 = 2×137	8 \times 37
8 \times 29	9 \times 28	275 = 5×55	297 = 3×99
233 Prime	12 \times 21	11 \times 25	9 \times 33
234 = 2×117	14 \times 18	276 = 2×138	11 \times 27
3 \times 78	253 = 11×23	3 \times 92	298 = 2×149
6 \times 39	254 = 2×127	4 \times 69	299 = 13×23
9 \times 26	255 = 3×85	6 \times 46	300 = 2×150
13 \times 18	5 \times 51	12 \times 23	3 \times 100
235 = 5×47	15 \times 17	277 Prime	4 \times 75
236 = 2×118	256 = 2×128	278 = 2×139	5 \times 60
4 \times 59	4 \times 64	279 = 3×93	6 \times 50
237 = 3×79	8 \times 32	9 \times 31	10 \times 30
238 = 2×119	16 \times 16	280 = 2×140	12 \times 25
7 \times 34	257 Prime	4 \times 70	15 \times 20
14 \times 17	258 = 2×129	5 \times 56	301 = 7×43
239 Prime	3 \times 86	7 \times 40	302 = 2×151
	6 \times 43	8 \times 35	303 = 3×101
	259 = 7×37	10 \times 28	304 = 2×152
	260 = 2×130	14 \times 20	4 \times 76
	4 \times 65	281 Prime	8 \times 38
	5 \times 52	282 = 2×141	16 \times 19
	10 \times 26	3 \times 94	305 = 5×61
	13 \times 20	6 \times 47	

Table IV (Continued)

306 = 2×153	326 = 2×163	348 = 2×174	368 = 2×184
3 × 102	327 = 3×109	3 × 116	4 × 92
6 × 51	328 = 2×164	4 × 87	8 × 46
9 × 34	4 × 82	6 × 58	16 × 23
17 × 18	8 × 41	12 × 29	369 = 3×123
307 Prime	329 = 7×47	349 Prime	9 × 41
308 = 2×154	330 = 2×165	350 = 2×175	370 = 2×185
4 × 77	3 × 110	5 × 70	5 × 74
7 × 44	5 × 66	7 × 50	10 × 37
11 × 28	6 × 55	10 × 35	371 = 5×53
14 × 22	10 × 33	14 × 25	372 = 2×186
309 = 3×103	11 × 30	351 = 3×117	3 × 124
310 = 2×155	15 × 22	9 × 39	4 × 93
5 × 62	331 Prime	13 × 27	6 × 62
10 × 31	332 = 2×166	352 = 2×176	12 × 31
311 = Prime	4 × 83	4 × 88	373 Prime
312 = 2×156	333 = 3×111	8 × 44	374 = 2×187
3 × 104	9 × 37	11 × 32	11 × 34
4 × 78	334 = 2×167	16 × 22	17 × 22
6 × 52	335 = 5×67	353 Prime	375 = 3×125
8 × 39	336 = 2×168	354 = 2×177	5 × 75
12 × 26	3 × 112	3 × 118	15 × 25
13 × 24	4 × 84	6 × 59	376 = 2×188
313 Prime	6 × 56	355 = 5×71	4 × 94
314 = 2×157	7 × 48	356 = 2×178	8 × 47
315 = 3×105	8 × 42	4 × 89	377 = 13×29
5 × 63	12 × 28	357 = 3×119	378 = 2×189
7 × 45	14 × 24	7 × 51	3 × 126
9 × 35	16 × 21	17 × 21	6 × 63
15 × 21	337 Prime	358 = 2×179	7 × 54
316 = 2×158	338 = 2×169	359 Prime	9 × 42
4 × 79	13 × 26	360 = 2×180	14 × 27
317 Prime	339 = 3×113	3 × 120	18 × 21
318 = 2×159	340 = 2×170	4 × 90	379 Prime
3 × 106	4 × 85	5 × 72	380 = 2×190
6 × 53	5 × 68	6 × 60	4 × 95
319 = 11×29	10 × 34	8 × 45	5 × 76
320 = 2×160	17 × 20	9 × 40	10 × 38
4 × 80	341 = 11×31	10 × 36	19 × 20
5 × 64	342 = 2×171	12 × 30	381 = 3×127
8 × 40	3 × 114	15 × 24	382 = 2×191
10 × 32	6 × 57	18 × 20	383 Prime
16 × 20	9 × 38	361 = 19×19	384 = 2×192
321 = 3×107	18 × 19	362 = 2×181	3 × 128
322 = 2×161	343 = 7×49	363 = 3×121	4 × 96
7 × 46	344 = 2×172	11 × 33	6 × 64
14 × 23	4 × 86	364 = 2×182	8 × 48
323 = 17×19	8 × 43	4 × 91	12 × 32
324 = 2×162	345 = 3×115	7 × 52	16 × 24
3 × 108	5 × 69	13 × 28	385 = 5×77
4 × 81	15 × 23	14 × 26	7 × 55
6 × 54	346 = 2×173	365 = 5×73	11 × 35
9 × 36	347 Prime	366 = 2×183	386 = 2×193
12 × 27		3 × 122	387 = 3×129
18 × 18		6 × 61	9 × 43
325 = 5×65		367 Prime	388 = 2×194
13 × 25			4 × 97

Table IV (Continued)

389	Prime	408	$= 2 \times 204$	429	$= 3 \times 143$	448	$= 2 \times 224$
390	$= 2 \times 195$		3×136		11×39		4×112
	3×130		4×102		13×33		7×64
	5×78		6×68	430	$= 2 \times 215$		8×56
	6×65		8×51		5×86		14×32
	10×39		12×34		10×43		16×28
	13×30		17×24	431	Prime	449	Prime
	15×26	409	Prime	432	$= 2 \times 216$	450	$= 2 \times 225$
391	$= 17 \times 23$	410	$= 2 \times 205$		3×144		3×150
392	$= 2 \times 196$		5×82		4×108		5×90
	4×98		10×41		6×72		6×75
	7×56	411	$= 3 \times 137$		8×54		9×50
	8×49	412	$= 2 \times 206$		9×48		10×45
	14×28		4×103		12×36		15×30
393	$= 3 \times 131$	413	$= 7 \times 59$		16×27		18×25
394	$= 2 \times 197$	414	$= 2 \times 207$		18×24	451	$= 11 \times 41$
395	$= 5 \times 79$		3×138	433	Prime	452	$= 2 \times 226$
396	$= 2 \times 198$		6×69	434	$= 2 \times 217$		4×113
	3×132		9×46		7×62	453	$= 3 \times 151$
	4×99		18×23		14×31	454	$= 2 \times 227$
	6×66	415	$= 5 \times 83$	435	$= 3 \times 145$	455	$= 5 \times 91$
	9×44	416	$= 2 \times 208$		5×87		7×65
	11×36		4×104		15×29		13×35
	12×33		8×52	436	$= 2 \times 218$	456	$= 2 \times 228$
	18×22		13×32		4×109		3×152
397	Prime		16×26	437	$= 19 \times 23$		4×114
398	$= 2 \times 199$	417	$= 3 \times 139$	438	$= 2 \times 219$		6×76
399	$= 3 \times 133$	418	$= 2 \times 109$		3×146		8×57
	7×57		11×38		6×73		12×38
	19×21		19×22	439	Prime		19×24
400	$= 2 \times 200$	419	Prime	440	$= 2 \times 220$	457	Prime
	4×100	420	$= 2 \times 210$		4×110	458	$= 2 \times 229$
	5×80		3×140		5×88	459	$= 3 \times 153$
	8×50		4×105		8×55		9×51
	10×40		5×84		10×44		17×27
	16×25		6×70		11×40	460	$= 2 \times 230$
	20×20		7×60		20×22		4×115
401	Prime		10×42	441	$= 3 \times 147$		5×92
402	$= 2 \times 201$		12×35		7×63		10×46
	3×134		14×30		9×49		20×23
	6×67		15×28		21×21	461	Prime
403	$= 13 \times 31$		20×21	442	$= 2 \times 221$	462	$= 2 \times 231$
404	$= 2 \times 202$	421	Prime		13×34		3×154
	4×101	422	$= 2 \times 211$		17×26		6×77
405	$= 3 \times 135$	423	$= 3 \times 141$	443	Prime		7×66
	5×81		9×47	444	$= 2 \times 222$		11×42
	9×45	424	$= 2 \times 212$		3×148		14×33
	15×27		4×106		4×111		21×22
406	$= 2 \times 203$		8×53		6×74	463	Prime
	7×58	425	$= 5 \times 85$		12×37	464	$= 2 \times 232$
	14×29		17×25	445	$= 5 \times 89$		4×116
407	$= 11 \times 37$	426	$= 2 \times 213$	446	$= 2 \times 223$		8×58
			3×142	447	$= 3 \times 149$		16×29
			6×71			465	$= 3 \times 155$
		427	$= 7 \times 61$				5×93
		428	$= 2 \times 214$				15×31
			4×107			466	$= 2 \times 233$

Table IV (Continued)

467 = Prime	486 = 2×243	504 = 2×252	522 = 2×261
468 = 2×234	3 \times 162	3 \times 168	3 \times 174
3 \times 156	6 \times 81	4 \times 126	6 \times 87
4 \times 117	9 \times 54	6 \times 84	9 \times 58
6 \times 78	18 \times 27	7 \times 72	18 \times 29
9 \times 52	487 = Prime	8 \times 63	523 = Prime
12 \times 39	488 = 2×244	9 \times 56	524 = 2×262
13 \times 36	4 \times 122	12 \times 42	4 \times 131
18 \times 26	8 \times 61	14 \times 36	525 = 3×175
469 = 7×67	489 = 3×163	18 \times 28	5 \times 105
470 = 2×235	490 = 2×245	21 \times 24	7 \times 75
5 \times 94	5 \times 98	505 = 5×101	15 \times 35
10 \times 47	7 \times 70	506 = 2×253	21 \times 25
471 = 3×157	10 \times 49	11 \times 46	526 = 2×263
472 = 2×236	14 \times 35	22 \times 23	527 = 17×31
4 \times 118	491 = Prime	507 = 3×169	528 = 2×264
8 \times 59	492 = 2×246	13 \times 39	3 \times 176
473 = 11×43	3 \times 164	508 = 2×254	4 \times 132
474 = 2×237	4 \times 123	4 \times 127	6 \times 88
3 \times 158	6 \times 82	509 = Prime	8 \times 66
6 \times 79	12 \times 41	510 = 2×255	11 \times 48
475 = 5×95	493 = 17×29	3 \times 170	12 \times 44
19 \times 25	494 = 2×247	5 \times 102	16 \times 33
476 = 2×238	13 \times 38	6 \times 85	22 \times 24
4 \times 119	19 \times 26	10 \times 51	529 = 23×23
7 \times 68	495 = 3×165	15 \times 34	530 = 2×265
14 \times 34	5 \times 99	17 \times 30	5 \times 106
17 \times 28	9 \times 55	511 = 7×73	10 \times 53
477 = 3×159	11 \times 45	512 = 2×256	531 = 3×177
9 \times 53	15 \times 33	4 \times 128	9 \times 59
478 = 2×238	496 = 2×298	8 \times 64	532 = 2×266
479 = Prime	4 \times 124	16 \times 32	4 \times 133
480 = 2×240	8 \times 62	513 = 3×171	7 \times 76
3 \times 160	16 \times 31	9 \times 57	14 \times 38
4 \times 120	497 = 7×71	19 \times 27	19 \times 28
5 \times 96	498 = 2×299	514 = 2×257	533 = 13×41
6 \times 80	3 \times 166	515 = 5×103	534 = 2×267
8 \times 60	6 \times 83	516 = 2×258	3 \times 178
10 \times 48	499 = Prime	3 \times 172	6 \times 89
12 \times 40	500 = 2×250	4 \times 129	535 = 5×107
15 \times 32	4 \times 125	6 \times 86	536 = 2×268
16 \times 30	5 \times 100	12 \times 43	4 \times 134
20 \times 24	10 \times 50	517 = 11×47	8 \times 67
481 = 13×37	20 \times 25	518 = 2×259	537 = 3×179
482 = 2×241	501 = 3×167	7 \times 74	538 = 2×269
483 = 3×161	502 = 2×251	14 \times 37	539 = 7×77
7 \times 69	503 = Prime	519 = 3×173	11 \times 49
21 \times 23		520 = 2×260	
484 = 2×242		4 \times 130	
4 \times 121		5 \times 104	
11 \times 44		8 \times 65	
22 \times 22		10 \times 52	
485 = 5×97		13 \times 40	
		20 \times 26	
		521 = Prime	

Table IV (Continued)

540 = 2×270	558 = 2×279	576 = 2×288	594 = 2×297
3×180	3×186	3×192	3×198
4×135	6×93	4×144	6×99
5×108	9×62	6×96	9×66
6×90	18×31	8×72	11×54
9×60	559 = 13×43	9×64	18×33
10×54	560 = 2×280	12×48	22×27
12×45	4×140	16×36	595 = 5×119
15×36	5×112	18×32	7×85
18×30	7×80	24×24	17×35
20×27	8×70	577 Prime	596 = 2×298
541 Prime	10×56	578 = 2×289	4×149
542 = 2×271	14×40	17×34	597 = 3×199
543 = 3×181	16×35	579 = 3×193	598 = 2×299
544 = 2×272	20×28	580 = 2×290	13×46
4×136	561 = 3×187	4×145	23×26
8×68	11×51	5×116	599 Prime
16×34	17×33	10×58	600 = 2×300
17×32	562 = 2×281	20×29	3×200
545 = 5×109	563 Prime	581 = 7×83	4×150
546 = 2×273	564 = 2×282	582 = 2×291	5×120
3×182	3×188	3×194	6×100
6×91	4×141	6×97	8×75
7×78	6×94	583 = 11×53	10×60
13×42	12×47	584 = 2×292	12×50
14×39	565 = 5×113	4×146	15×40
21×26	566 = 2×283	8×73	20×30
547 Prime	567 = 3×189	585 = 3×195	24×25
548 = 2×274	7×81	5×117	601 Prime
4×137	9×63	9×65	602 = 2×301
549 = 3×183	21×27	13×45	7×86
9×61	568 = 2×284	15×39	14×43
550 = 2×275	4×142	586 = 2×293	603 = 3×201
5×110	8×71	587 Prime	9×67
10×55	569 Prime	588 = 2×294	604 = 2×302
11×50	570 = 2×285	3×196	4×151
22×25	3×190	4×147	605 = 5×121
551 = 19×29	5×114	6×98	11×55
552 = 2×276	6×95	7×84	606 = 2×303
3×184	10×57	12×49	3×202
4×138	15×38	14×42	6×101
6×92	19×30	21×28	607 Prime
8×69	571 Prime	589 = 19×31	608 = 2×304
12×46	572 = 2×286	590 = 2×295	1×152
23×24	4×143	5×118	8×76
553 = 7×79	11×52	10×59	16×38
554 = 2×277	13×44	591 = 3×197	19×32
555 = 3×185	22×26	592 = 2×296	609 = 3×203
5×111	573 = 3×191	4×148	7×87
15×37	574 = 2×287	8×74	21×29
556 = 2×278	7×82	16×37	610 = 2×305
4×139	14×41	593 Prime	5×122
557 Prime	575 = 5×115		10×61
	23×25		611 = 13×47

Table IV (Concluded)

612 = 2 × 306	616 = 2 × 308	619 Prime	624 = 2 × 312
3 × 204	4 × 154	620 = 2 × 310	3 × 208
4 × 152	7 × 88	4 × 155	4 × 156
6 × 102	8 × 77	5 × 124	6 × 104
9 × 68	11 × 56	10 × 62	8 × 78
12 × 51	14 × 44	20 × 31	12 × 52
17 × 36	22 × 28	621 = 3 × 207	13 × 48
18 × 34	617 Prime	9 × 69	16 × 39
613 Prime	618 = 2 × 309	23 × 27	24 × 26
614 = 2 × 307	3 × 206	622 = 2 × 311	625 = 5 × 125
615 = 3 × 205	6 × 103	623 = 7 × 89	25 × 25
5 × 123			
15 × 41			

ANSWERS

The references at the head of each section are to the numbers of the exercises.

No. 1				
	30	70	69	53
	86	54	25	109
1. 32	42	110	81	65
2. 30	98	66	37	21
3. 29	26	22	93	77
4. 29	82	78	49	40
5. 29	38	34	105	96
6. 31	94	90	68	52
7. 31	50	53	24	108
8. 18	106	109	80	64
9. 37	62	65	36	48
10. 31	25	21	92	104
11. 25	81	77	20	60
12. 35	37	61	76	16
13. 34	93	17	32	72
14. 29	49	73	88	28
15. 26	105	29	44	84
16. 25	33	85	100	47
17. 30	89	41	56	103
18. 33	45	97	19	59
19. 27	101	60	75	15
20. 30	57	16	31	71
21. 33	13	72	87	55
22. 26	69	28	43	111
23. 28	32	84	99	67
24. 27	88		27	23
	44		83	79
	100	No. 3	39	35
No. 2	56		95	91
12	40	1. 59	51	54
68	96	2. 51	107	110
24	52	3. 56	63	66
80	108	4. 70	26	22
36	64	5. 62	82	78
92	20	6. 55	38	62
48	76	7. 57	94	18
104	39	8. 59	50	74
67	95	9. 53	106	30
23	51	10. 51	34	86
79	107	11. 69	90	42
35	63	12. 58	46	98
91	47	13. 60	102	61
19	103	14. 65	58	17
75	59	15. 59	14	73
31	15	16. 61	70	29
87	71	17. 53	33	85
43	27	18. 53	89	
99	83		45	
55	46		101	No. 5
18	102	No. 4	57	
74	58		41	14
	14	13	97	70

26	109	46	113	29
82	65	102	69	85
38	49	58	25	41
94	105	21	81	97
50	61	77	37	53
106	17	33	93	109
69	73	89	56	37
25	29	45	112	93
81	85	101	68	49
37	48	29	24	105
93	104	85	80	61
21	60	41	64	17
77	16	97	20	73
33	72	53	76	36
89	56	109	32	92
45	112	65	88	48
101	68	28	44	104
57	24	84	100	60
20	80	40	63	44
76	36	96	19	100
32	92	52	75	56
88	55	108	31	112
44	111	36	87	68
100	67	92	No. 7	24
28	23	48		80
84	79	104		43
40	63	60	16	99
96	19	16	72	55
52	75	72	28	111
108	31	35	84	67
64	87	91	40	51
27	43	47	96	107
83	99	103	52	63
39	62	59	108	19
95	18	43	71	75
51	74	99	27	31
107	30	55	83	87
35	86	111	39	50
91	No. 6	67	95	106
47		23	23	62
103		79	79	18
59	15	42	35	74
15	71	98	91	58
71	27	54	47	114
34	83	110	103	70
90	39	66	59	26
46	95	50	22	82
102	51	106	78	38
58	107	62	34	94
42	70	18	90	57
98	26	74	46	113
54	82	30	102	69
110	94	86	30	25
66	38	49	86	81
22	94	105	42	65
78	22	61	98	21
41	78	17	54	77
97	34	73	110	33
53	90	57	66	89

156 THE ART OF CALCULATION

45	37	30	113	98
101	93	86	69	26
64	49	42	53	82
20	105	98	109	38
76	61	54	65	94
32	45	110	21	50
88	101	73	77	106
	57	29	33	62
No. 8	113	85	89	25
(Same as	69	41	52	81
No. 1)	25	97	108	37
	81	25	64	93
No. 9	44	81	20	49
17	100	37	76	105
73	56	93	60	33
29	112	49	116	89
85	68	105	72	45
41	52	61	28	101
97	108	24	84	57
53	64	80	40	113
109	20	36	96	69
72	76	92	59	32
28	32	48	115	88
84	88	104	71	44
40	51	32	27	100
96	107	88	83	56
24	63	44	67	112
80	19	100	23	40
36	75	56	79	96
92	59	112	35	52
48	115	68	91	108
104	71	31	47	64
60	27	87	103	20
23	83	43	66	76
79	39	99	22	39
35	95	55	78	95
91	58	111	34	51
47	114	39	90	107
103	70	95		63
31	26	51		47
87	82	107	No. 11	103
43	66	63	(Same as	59
99	22	19	No. 9)	115
55	78	75		71
111	34	38	No. 12	27
67	90	94		83
30	46	50		46
86	102	106	19	102
42	65	62	75	58
98	21	46	31	114
54	77	102	87	70
110	33	58	43	54
38	89	114	99	110
94		70	55	66
50		26	111	22
106	No. 10	82	74	78
62		45	30	34
18	18	101	86	90
74	74	57	42	53

109	No. 14	84	14. 656	61
65		47	15. 858	117
21	20	103		73
77	76	59		29
61	32	115	No. 16	85
117	88	71		48
73	44	55	21	104
29	100	111	77	60
85	56	67	33	116
41	112	23	89	72
97	75	79	45	56
60	31	35	101	112
116	87	91	57	68
72	43	54	113	24
28	99	110	76	80
84	27	66	32	36
68	83	22	88	92
24	39	78	44	55
80	95	62	100	111
36	51	118	28	67
92	107	74	84	23
48	63	30	40	79
104	26	86	96	63
67	82	42	52	119
23	38	98	108	75
79	94	61	64	31
35	50	117	27	87
91	106	73	83	43
	34	29	39	99
	90	85	95	62
	46	69	51	118
No. 13	102	25	107	74
	58	81	35	30
1. 365	114	37	91	86
2. 268	70	93	47	70
3. 371	33	49	103	26
4. 433	89	105	59	82
5. 257	45	68	115	38
6. 327	101	24	71	94
7. 209	57	80	34	50
8. 270	113	36	90	106
9. 287	41	92	46	69
10. 410	97		102	25
11. 257	53		58	81
12. 404	109	No. 15	114	37
13. 231	65	1. 620	42	93
14. 217	21	2. 777	98	
15. 311	77	3. 716	54	
16. 303	40	4. 562	110	No. 17
17. 254	96	5. 432	66	1. 1059
18. 237	52	6. 590	22	2. 1055
19. 308	108	7. 624	78	3. 903
20. 343	64	8. 716	41	4. 963
21. 350	48	9. 885	97	5. 897
22. 360	104	10. 828	53	6. 1113
23. 308	60	11. 424	109	7. 1067
24. 271	116	12. 592	65	8. 759
25. 341	72	13. 535	49	9. 994
	28		105	

158 THE ART OF CALCULATION

10. 932	118	11. 7	93	88
	74	12. 34	49	72
No. 18	30	13. 52	105	28
22	86	14. 11	61	84
78	49	15. 52	117	40
34	105		73	96
90	61	No. 20	36	52
46	117		92	108
102	73	1. 28	48	71
58	57	2. 28	104	27
114	113	3. 12	60	83
77	69	4. 19	116	39
33	25	5. 15	44	95
89	81	6. 26	100	
45	37	7. 19	56	No. 22
101	93	8. 18	112	
29	56	9. 48	68	1. 294
85	112	10. 21	24	2. 234
41	68	11. 39	80	3. 414
97	24	12. 17	43	4. 358
53	80	13. 26	99	5. 379
109	64	14. 58	55	6. 381
65	120	15. 28	111	7. 370
28	76	16. 18	67	8. 347
84	32	17. 29	51	9. 221
40	88	18. 19	107	10. 374
96	44	19. 29	63	
52	100		119	
108	63	No. 21	75	
36	119		31	
92	75	23	87	No. 23
48	31	79	50	
104	87	35	106	
60	71	91	62	
116	27	47	118	1. 521
72	83	103	74	2. 213
35	39	59	58	3. 233
91	95	115	114	4. 321
47	51	78	70	5. 331
103	107	34	26	6. 313
59	70	90	82	7. 252
115	26	46	38	8. 412
43	82	102	94	9. 212
99	38	30	57	10. 130
55	94	86	113	11. 122
111		42	69	12. 441
67	No. 19	98	25	13. 432
23	1. 12	54	81	14. 351
79	2. 34	110	65	15. 221
42	3. 21	66	121	
98	4. 56	29	77	No. 24
54	5. 33	85	33	
110	6. 78	41	89	
66	7. 12	97	45	24
50	8. 13	53	101	80
106	9. 12	109	64	36
62	10. 21	37	120	92
			76	48
			32	

104	115	31	91	22. 437
60	71	87	47	23. 722
116	27	43	103	24. 109
79	83	99	66	25. 515
35	39	55	122	26. 209
91	95	111	78	27. 336
47	58	39	34	28. 107
103	114	95	90	29. 868
31	70	51	74	30. 419
87	26	107	30	
43	82	63	86	
99	66	119	42	No. 28
55	122	75	98	26
111	78	38	54	82
67	34	94	110	38
30	90	50	73	94
86	46	106	29	50
42	102	62	85	106
98	65	118	41	62
54	121	46	97	118
110	77	102		81
38	33	58		37
94	89	114	No. 26	93
50	73	70	1. \$655.71	49
106	29	26	2. \$751.32	105
62	85	82	3. \$604.24	33
118	41	45	4. \$577.21	89
74	97	101	5. \$718.69	45
37	53	57	6. \$769.64	101
93	109	113	7. \$488.04	57
49	72	69	8. \$691.93	113
105	28	53		69
61	84	109		32
117	40	65	No. 27	88
45	96	121	1. 215	44
101		77	2. 415	100
57	No. 25	33	3. 209	56
113	25	89	4. 329	112
69	81	52	5. 778	40
25	37	108	6. 109	96
81	93	64	7. 214	52
44	49	120	8. 248	108
100	105	76	9. 128	64
56	61	60	10. 237	120
112	117	116	11. 403	76
68	80	72	12. 106	39
52	36	28	13. 125	95
108	92	84	14. 125	51
64	48	40	15. 136	107
120	104	96	16. 204	63
76	32	59	17. 109	119
32	88	115	18. 143	47
88	51	71	19. 107	103
51	107	83	20. 308	59
107	63	67	21. 309	115
119	56	123		71
75	112	79		27
59	68	35		

83	83	110	35	118
46	39	66	91	74
102	95	122	47	30
58	51	78	103	86
114	107	62	59	70
70	63	118	115	126
54	119	74	71	82
110	82	30	34	38
66	38	86	90	94
122	94	42	46	50
78	50	98	102	106
34	106	61	58	69
90	34	117	114	125
53	90	73	42	81
109	46	29	98	37
65	102	85	54	93
121	58	69	110	79
77	114	125	66	33
61	70	81	112	89
117	33	37	78	45
73	89	93	41	101
29	45	49	97	57
85	101	105	53	113
41	57	68	109	76
97	113	124	65	32
60	41	80	121	88
116	97	36	49	44
72	53	92	105	100
28	109	76	61	
84	65	32	117	
68	121	88	73	
124	77	44	29	No. 31
80	40	100	85	1. 621
36	96	56	48	2. 585
92	52	112	104	3. 687
48	108	75	60	4. 647
104	54	31	116	5. 630
67	120	86	72	6. 605
123	48	43	56	7. 570
79	104	99	112	8. 671
35	60		68	9. 625
91	116		124	10. 624
75	72	No. 30	80	
31	28		36	
87	84		92	
43	47	28	55	
99	103	84	111	No. 32
55	59	40	67	1. 161
111	115	96	123	2. 292
74	71	52	79	3. 71
30	55	108	63	4. 191
86	111	64	119	5. 171
42	67	120	75	6. 64
98	123	83	31	7. 252
	79	39	87	8. 197
No. 29	35	95	43	9. 623
	91	51	99	10. 284
27	64	107	62	

11. 94
12. 387
13. 170
14. 61
15. 593
16. 195
17. 394
18. 295
19. 492
20. 681

No. 33

1. 465
2. 579
3. 164
4. 186
5. 153
6. 48
7. 489
8. 186
9. 488
10. 377
11. 329
12. 469
13. 288
14. 56
15. 216
16. 184
17. 249
18. 77
19. 289
20. 169

No. 34

1. \$995.69
2. \$1044.85
3. \$954.07
4. \$1002.63
5. \$994.32
6. \$897.80
7. \$1122.66
8. \$1051.42

No. 35

1. 395
2. 297
3. 92
4. 299
5. 298
6. 195
7. 298
8. 399
9. 494

10. 497
11. 296
12. 94
13. 495
14. 294
15. 299
16. 198
17. 197
18. 397
19. 293
20. 692
21. 198
22. 294
23. 596
24. 99
25. 395

No. 36

1. 985
2. 987
3. 975
4. 1008
5. 953
6. 1011
7. 1042
8. 1032
9. 1095
10. 1012

No. 37

1. 347
2. 189
3. 349
4. 78
5. 107
6. 259
7. 189
8. 119
9. 66
10. 88
11. 215
12. 178
13. 178
14. 9
15. 227
16. 109
17. 114
18. 249
19. 234
20. 29
21. 298
22. 284
23. 38
24. 378
25. 129

No. 38

1. \$42357.49
2. \$57112.34
3. \$54738.19
4. \$62369.15
5. \$70468.35
6. \$63801.69

No. 39

1. \$4.35
2. \$5.59
3. \$.94
4. \$1.48
5. \$6.92
6. \$7.63
7. \$2.31
8. \$6.84
9. \$3.70
10. \$2.76
11. \$2.29
12. \$6.76
13. \$3.59
14. \$5.96
15. \$1.56
16. \$3.89
17. \$2.68
18. \$6.92
19. \$3.49
20. \$5.97

No. 40

(Same as
No. 15)

No. 41

1. \$95513.02
2. \$102635.78
3. \$98506.46
4. \$117398.69
5. \$95153.78
6. \$99073.91

No. 42

(Same as
No. 39)

No. 43

1. \$.93
2. \$1.20

3. \$2.81
4. \$.65
5. \$1.96
6. \$5.84
7. \$2.95
8. \$1.65
9. \$2.24
10. \$.71
11. \$1.89
12. \$.73
13. \$1.23
14. \$1.63
15. \$1.71
16. \$2.48
17. \$1.86
18. \$1.94
19. \$2.45
20. \$1.63

No. 44

(Same as
No. 45)

No. 45

2
114
26
138
50
162
74
186
112
24
136
48
160
16
128
40
152
64
176
88
14
126
38
150
62
174
30
142
54
166
78

162 THE ART OF CALCULATION

190	124	174	228	336
102	36	63	52	160
28	148	231	276	384
140	60	99	100	208
52	172	267	324	144
164	98	135	148	368
76	10	87	372	192
188	122	255	224	16
44	34	123	48	240
156	146	291	272	64
68		159	96	288
180	No. 46	27	320	140
92		195	32	364
4	3	84	256	188
116	171	252	80	12
42	39	120	304	236
154	207	288	128	172
66	75	156	352	396
178	243	108	176	220
90	111	276	28	44
58	279	144	252	268
170	168	12	76	92
82	36	180	300	316
194	204	48	124	168
106	72	216	348	392
18	240	105	60	216
130	24	273	284	40
56	192	141	108	264
168	60	9	332	200
80	228	177	156	24
192	96	129	380	248
104	264	297	204	72
72	132	165	56	292
184	21	33	280	120
96	189	201	104	344
8	57	69	328	196
120	225	237	152	20
32	93	126	376	244
144	261	294	88	68
70	45	162	312	296
182	213	30	136	
94	81	198	360	
6	249	150	184	No. 48
118	117	18	8	
86	285	186	232	1. \$3433540.07
198	153	54	84	2. \$2509179.07
110	42	222	308	3. \$3688667.60
22	210	90	132	4. \$3251326.81
134	78	258	356	5. \$3449296.55
46	246	147	180	6. \$3353169.99
158	114	15	116	
84	282	183	340	
196	66	51	164	No. 49
108	234	219	388	
20	102		212	1. \$18.53
132	270	No. 47	36	2. \$25.66
100	138		260	3. \$23.95
12	6	4	112	4. \$14.78

5. \$41.76	170	No. 51	174	259
6. \$38.38	450		510	651
7. \$15.74	230	(Same as	246	392
8. \$42.95	10	No. 49)	582	84
9. \$60.76	290		318	476
10. \$71.19	105	No. 52	54	168
11. \$66.57	385		390	560
12. \$59.85	165	6	168	56
13. \$93.72	445	342	504	448
14. \$80.90	225	78	240	140
15. \$75.68	145	414	576	532
16. \$61.52	425	150	312	224
	205	486	216	616
	485	222	552	308
	265	558	288	49
No. 50	45	336	24	441
	325	72	360	133
5	140	408	96	525
285	420	144	432	217
65	200	480	210	609
345	480	48	546	105
125	260	384	282	497
405	180	120	18	189
185	460	456	354	581
465	240	192	258	273
280	20	528	594	665
60	300	264	330	357
340	80	42	66	98
120	360	378	402	490
400	175	114	138	182
40	455	450	474	574
320	235	186	252	266
100	15	522	588	658
380	295	90	324	154
160	215	426	60	546
440	495	162	396	238
220	275	498	300	630
35	55	234	36	322
315	335	570	372	14
95	115	306	108	406
375	395	84	444	147
155	210	420	180	539
435	490	156	516	231
75	270	492	294	623
355	50	228	30	315
135	330	564	366	203
415	250	132	102	595
195	30	468	438	287
475	310	204		679
255	90	540		371
70	370	276	No. 53	63
350	150	12		455
130	430	348	7	196
410	245	126	399	588
190	25	462	91	280
470	305	198	483	672
110	85	534	175	364
390	365	270	567	252

164 THE ART OF CALCULATION

644	12. \$55.60	712	No. 59	639
336	13. \$97.15	360		243
28	14. \$73.69	232	1. 795	747
420	15. \$61.63	680	2. 682	351
112	16. \$68.20	328	3. 564	855
504		776	4. 814	459
245		424	5. 598	126
637	No. 56	72	6. 924	630
329		520	7. 810	234
21	8	224	8. 946	738
413	456	672	9. 1032	342
301	104	320	10. 912	846
693	552	768	11. 901	198
385	200	416	12. 621	702
77	648	288	13. 665	306
469	296	736	14. 308	810
161	744	384	15. 962	414
553	448	32	16. 714	18
294	96	480	17. 1008	522
686	544	128	18. 364	189
378	192	576	19. 736	693
70	640	280	20. 782	297
462	64	728	21. 855	801
350	512	376	22. 864	405
42	160	24	23. 865	261
434	608	472	24. 988	765
126	256	344	25. 667	369
518	704	792		873
210	352	440		477
602	56	88	No. 60	81
343	504	536		585
35	152	184	9	252
427	600	632	513	756
119	248	336	117	360
511	696	784	621	864
	120	432	225	468
	568	80	729	324
	216	528	333	828
	664	400	837	432
	312	48	504	36
	760	496	108	540
	408	144	612	144
	112	592	216	648
	560	240	720	315
	208	688	72	819
	656	392	572	423
	304	40	180	27
	752	488	684	531
	176	136	288	387
	624	584	792	891
	272		396	495
	720		63	99
	368	No. 57	567	603
	16	(Same as	171	207
	464	No. 15)	675	711
	168	No. 58	279	378
	616	(Same as	783	882
	264	No. 55)	135	486
1. \$6537136.94				
2. \$6295852.28				
3. \$6328194.91				
4. \$5945296.77				
	No. 55			
1. \$19.76				
2. \$18.86				
3. \$44.51				
4. \$26.39				
5. \$41.42				
6. \$6.20				
7. \$12.22				
8. \$19.63				
9. \$87.27				
10. \$84.51				
11. \$71.61				

90	374	No. 62	2. \$836.87
594	990		3. \$666.99
450	506	1. \$11230083.55	4. \$829.97
54	22	2. \$10797546.08	5. \$634.22
558	608	3. \$8876665.99	6. \$827.43
162	231	4. \$8230948.08	7. \$857.76
666	847		8. \$527.72
270	363	No. 63	9. \$418.44
774	979		10. \$906.92
441	495	1. \$47.65	11. \$447.71
45	319	2. \$6.21	12. \$586.87
549	935	3. \$79.61	13. \$407.46
153	451	4. \$34.74	14. \$510.63
657	1067	5. \$14.68	15. \$533.62
No. 61	583	6. \$27.74	16. \$663.85
11	99	7. \$27.93	No. 68
627	715	8. \$21.85	(Same as No. 17)
143	308	9. \$54.46	
759	924	10. \$13.83	No. 69
275	440	11. \$36.49	(Same as No. 67)
891	1056	12. \$4.46	
407	572	13. \$50.47	
1023	396	14. \$8.53	
616	1012	15. \$27.16	
132	528	16. \$39.87	
748	44	No. 65	No. 71
264	660	(Same as No. 63)	1. \$276.69
880	176		2. \$855.51
88	792	No. 66	3. \$682.90
704	385		4. \$520.36
220	1001	1. 1827	5. \$773.79
836	517	2. 1705	6. \$891.54
352	33	3. 1170	7. \$326.93
968	649	4. 1376	8. \$245.59
484	473	5. 2511	9. \$371.93
77	1089	6. 2624	10. \$471.54
693	605	7. 3772	11. \$386.88
209	121	8. 1200	12. \$330.44
825	737	9. 1537	13. \$878.62
341	253	10. 1235	14. \$696.89
957	869	11. 1408	15. \$770.20
165	462	12. 1428	16. \$674.87
781	1078	13. 1407	No. 72
297	594	14. 1408	(Same as No. 22)
913	110	15. 2016	
429	726	16. 2418	No. 73
1045	550	17. 3772	
561	66	18. 1164	
154	682	19. 2015	
770	198	20. 2592	
286	814	No. 67	
902	330	1. \$846.98	
418	946		
1034	539		
242	55		
858	671		
	187		
	803		

166 THE ART OF CALCULATION

5. 3674994324
6. 1167178458
7. 1236433047
8. 6091457406
9. 1690209807
10. 1752668607
11. 1511041308
12. 3675686802
13. 1306128921
14. 1031412036
15. 1442533509

No. 74

1. 1536
2. 4606
3. 2646
4. 1495
5. 5313
6. 3230
7. 7347
8. 4814
9. 4284
10. 1295
11. 6624
12. 1624
13. 1886
14. 3618
15. 5494
16. 3861
17. 3344
18. 8608
19. 1612
20. 2655

No. 75

(Same as No. 71)

No. 76

(Same as No. 26)

No. 77

12
684
156
828
300
972
444
1116
672

144
816
288
960
96
768
240
912
384
1056
528
84
756
228
900
372
1044
180
852
324
996
468
1140
612
168
840
312
984
456
1128
264
936
408
1080
552
24
696
252
924
396
1068
540
348
1020
492
1164
636
108
780
336
1008
480
1152
624
432
1104
576
48

720
192
864
420
1092
564
36
708
516
1188
660
132
804
276
948
504
1176
648
120
792
600
72
744
216
888
360
1032
588
60
732
204
876

No. 78

(Same as No. 34)

No. 79

1. \$451.84
2. \$189.86
3. \$343.97
4. \$352.59
5. \$188.21
6. \$145.71
7. \$291.97
8. \$664.63
9. \$136.68
10. \$86.14
11. \$440.45
12. \$221.48
13. \$196.63
14. \$146.23
15. \$586.21
16. \$568.49

No. 80

1. 17081

2. 13361
3. 25543
4. 22632
5. 37893
6. 34323
7. 52643
8. 45201
9. 68302
10. 62693
11. 19602
12. 12312
13. 77922
14. 33033
15. 25662
16. 12831
17. 16086
18. 20274
19. 22263
20. 47583
21. 44896

No. 81

1. 123782280
2. 123895704
3. 135014592
4. 135128016
5. 601943392
6. 191177264
7. 202520776
8. 997746448
9. 276846856
10. 287077256
11. 247500064
12. 602056816
13. 213936568
14. 168939488
15. 236278872

No. 82

(Same as No. 38)

No. 83

1. \$451.84
2. \$189.86
3. \$343.97
4. \$352.59
5. \$188.21
6. \$145.71
7. \$291.97
8. \$664.63
9. \$136.68
10. \$86.14
11. \$440.45

12. \$221.48
13. \$196.63
14. \$146.23
15. \$586.21
16. \$568.49

No. 84

1. 19584
2. 23793
3. 28288
4. 24466
5. 17344
6. 21483
7. 24208
8. 21346
9. 25164
10. 12691
11. 17138
12. 21918
13. 30702
14. 36206
15. 33355
16. 17199
17. 27846
18. 31003
19. 29120
20. 33948
21. 16238

No. 86

1. \$95513.02
2. \$102635.78
3. \$98506.46
4. \$117398.69
5. \$95153.78
6. \$99073.91

No. 89

1. 170810
2. 133610
3. 255430
4. 226320
5. 378930
6. 343230
7. 526430
8. 452010
9. 683020
10. 626930
11. 196020
12. 123120
13. 779220
14. 330330

15. 256620
16. 128310
17. 160860
18. 202740
19. 222630
20. 465830
21. 448960

No. 90

- 13
- 741
- 169
- 897
- 325
- 1053
- 481
- 1209
- 728
- 156
- 884
- 312
- 1040
- 104
- 832
- 260
- 988
- 416
- 1144
- 572
- 91
- 819
- 247
- 975
- 403
- 1131
- 195
- 923
- 351
- 1079
- 507
- 1235
- 663
- 182
- 910
- 338
- 1066
- 494
- 1222
- 286
- 1014
- 442
- 1170
- 598
- 26
- 754
- 273

- 1001
- 429
- 1157
- 585
- 377
- 1105
- 533
- 1261
- 689
- 117
- 845
- 364
- 1092
- 520
- 1248
- 676
- 468
- 1196
- 624
- 52
- 780
- 208
- 936
- 455
- 1183
- 611
- 39
- 767
- 559
- 1287
- 715
- 143
- 871
- 299
- 1027
- 546
- 1274
- 702
- 130
- 858
- 650
- 78
- 806
- 234
- 962
- 390
- 1118
- 637
- 65
- 793
- 221
- 949

No. 91

(Same as No. 48)

No. 93

1. 195840
2. 237930
3. 282880
4. 244660
5. 173440
6. 214830
7. 242080
8. 213460
9. 251640
10. 126910
11. 171380
12. 219180
13. 307020
14. 362060
15. 333550
16. 171990
17. 278460
18. 310030
19. 291200
20. 339480
21. 162380

No. 94

1. 135025095
2. 135148821
3. 147377608
4. 147401334
5. 656616308
6. 208541386
7. 220915199
8. 1088369102
9. 301992119
10. 303151719
11. 269979836
12. 656740034
13. 233367857
14. 184383812
15. 257739453

No. 95

(Same as No. 54)

No. 97

1. 11211
2. 24642
3. 40051
4. 57902
5. 77691
6. 92412
7. 29432

19. 430265	No. 118	435	No. 123
20. 247275		1275	1. 157510725
21. 575276	(Same as No. 38)	615	2. 157655055
		1455	3. 171803640
		795	4. 171947970
		135	5. 765962140
		975	6. 243269630
No. 110	No. 119	420	7. 257704045
1. 146267910	15	1260	8. 1269714410
2. 146401938	855	600	9. 352282645
3. 159540624	195	1440	10. 365300645
4. 159674652	1035	780	11. 314939380
5. 711289224	375	540	12. 766106470
6. 225905508	1215	1380	13. 272230435
7. 239309622	555	720	14. 214972460
8. 1178991756	1395	60	15. 300660615
9. 327137382	840	900	
10. 339226182	180	240	No. 124
11. 292459608	1020	1080	(Same as No. 54)
12. 711423252	360	525	
13. 252799146	1200	1365	No. 126
14. 199628136	120	705	(Same as No. 62)
15. 279200034	960	45	
	300	885	No. 128
	1140	645	(Same as No. 58)
	480	1485	
No. 111	1320	825	No. 131
(Same as No. 26)	660	165	
	105	1005	
	945	345	No. 128
No. 113	285	1185	(Same as No. 58)
	1125	630	
1. 164232	465	1470	
2. 227238	1305	810	
3. 301464	225	150	
4. 377910	1065	990	
5. 456576	405	750	
6. 497502	1245	90	
7. 658752	585	930	
8. 172104	1425	270	
9. 243320	765	1110	
10. 279396	210	450	
11. 354252	1050	1290	
12. 427652	390	735	
13. 484432	1230	75	
14. 588078	570	915	
15. 671944	1410	255	
16. 175392	330	1095	
17. 173514	1170		
18. 257237	510		
19. 341968	1350		
20. 429525	690		
21. 519302	30		
	870	No. 120	
	315	(Same as No. 41)	
	1155	No. 122	
No. 115	495	(Same as No. 48)	
(Same as No. 34)	1335		
	675		

170 THE ART OF CALCULATION

304	368	340	51
1200	1264	1292	1003
496	672	544	731
1392	1568	1496	1683
240	864	748	935
1136	160	119	187
432	1056	1071	1139
1328	800	323	391
624	96	1275	1343
1520	992	527	714
816	288	1479	1666
224	1184	255	918
1120	480	1207	170
416	1376	459	1122
1312	784	1411	850
608	80	663	102
1504	976	1615	1054
352	272	867	306
1248	1168	238	1258
544		1190	510
1440		442	1462
736	No. 132	1394	833
32		646	85
928	1. 168753540	1598	1037
336	2. 168908172	374	289
1232	3. 184066656	1326	1241
528	4. 184221288	578	
1424	5. 820635056	1530	No. 141
720	6. 260633752	782	1. 179996355
464	7. 276098468	34	2. 180161289
1360	8. 1360237064	996	3. 196329672
656	9. 377427908	357	4. 196494606
1552	10. 391375108	1309	5. 875307972
848	11. 337419152	561	6. 277997874
144	12. 820789688	1513	7. 294492891
1040	13. 291661724	765	8. 1450859718
448	14. 230316784	493	9. 402573171
1344	15. 322121196	1445	10. 417449571
640		697	11. 359898924
1536		1649	12. 875472906
832	No. 140	901	13. 311093013
576		153	14. 245661108
1472		1105	15. 343581777
768	17	476	
64	969	1428	No. 148
960	221	680	18
256	1173	1632	1026
1152	425	884	234
560	1377	912	1242
1456	629	1564	450
752	1581	816	1458
48	952	68	666
944	204	1020	1674
688	1156	272	1008
1584	408	1224	
880	1360	595	
176	136	1547	
1072	1088	799	

216	1080	247	760
1224	288	1311	1824
432	1296	475	988
1440	630	1539	684
144	1638	703	1748
1152	846	1767	912
360	54	1064	76
1368	1062	228	1140
576	774	1292	304
1584	1782	456	1368
792	990	1520	665
126	198	152	1729
1134	1206	1216	893
342	414	380	57
1350	1422	1444	1121
558	756	608	817
1566	1764	1672	1881
270	972	836	1045
1278	180	133	209
486	1188	1197	1273
1494	900	361	437
702	108	1425	1501
1710	1116	589	798
918	324	1653	1862
252	1332	285	1026
1260	540	1349	190
468	1548	513	1254
1476	882	1577	950
684	90	741	114
1692	1098	1805	1178
396	306	969	342
1404	1314	266	1406
612		1330	570
1620		494	1634
828	No. 149	1558	931
36		722	95
1044	1. 191239170	1786	1159
378	2. 191414406	418	323
1386	3. 208592688	1482	1387
594	4. 208767924	646	
1602	5. 929980808	1710	
810	6. 295361996	874	
522	7. 312887314	38	
1530	8. 1541482372	1102	
738	9. 427718434	399	
1746	10. 443524034	1463	
954	11. 382378696	627	
162	12. 930156124	1691	
1170	13. 330524302	855	
504	14. 261005432	551	
1512	15. 365042358	1615	
720		779	
1728		1843	
936		1007	
648	No. 156	171	
1656		1235	
864	19	532	
72	1083	1596	
			No. 159
			1. 202481985
			2. 202667523
			3. 220855704
			4. 221041242
			5. 984653804
			6. 312726118
			7. 331281737
			8. 1632105026
			9. 452863697
			10. 469598497
			11. 404858468
			12. 984839342
			13. 349955591
			14. 276349756
			15. 386502939

172 THE ART OF CALCULATION

No. 165	180	13. 369386880	1785
20	1300	14. 291694080	861
1140	560	15. 407963520	2037
260	1680		1113
1380	800	No. 172	189
500	1920	21	1365
1620	1040	1197	588
740	720	273	1744
1860	1840	1449	840
1120	960	525	2016
240	80	1701	1092
1360	1200	777	756
480	320	1953	1932
1600	1440	1176	1008
180	700	252	84
1280	1820	1428	1260
400	940	504	336
1520	60	1680	1512
640	1180	168	735
1760	860	1344	1911
880	1980	420	987
140	1100	1596	63
1260	220	672	1239
380	1340	1848	903
1500	460	924	2079
620	1580	147	1155
1740	840	1323	231
300	1960	399	1407
1420	1080	1575	483
540	200	651	1659
1660	1320	1827	882
780	1000	315	2058
1900	120	1491	1134
1020	1240	567	210
280	360	1743	1386
1400	1480	819	1050
520	600	1995	126
1640	1720	1071	1302
760	980	294	378
1880	100	1470	1554
440	1220	546	630
1560	340	1722	1806
680	1460	798	1029
1800		1974	105
920	No. 166	462	1281
40	1. 213724800	1638	357
1160	2. 213920640	714	1533
420	3. 233118720	1890	
1540	4. 233314560	966	No. 173
660	5. 1039326720	42	1. 224967615
1780	6. 330090240	1218	2. 225173757
900	7. 349676160	441	3. 245381736
580	8. 1722727680	1617	4. 245587878
1700	9. 478008960	693	5. 1093999636
820	10. 495672960	1869	6. 347454362
1940	11. 427338240	945	7. 368070583
1060	12. 1039522560	609	

8.	1813350334	462
9.	503154223	1694
10.	521747423	726
11.	449818012	1958
12.	1094205778	990
13.	388818169	638
14.	307038404	1870
15.	429424101	902

No. 179

22	198
1254	1430
286	616
1518	1848
550	880
1782	2112
814	1144
2046	792
1232	2024
264	1056
1496	88
528	1320
1760	352
176	1584
1408	770
440	2002
1672	1034
704	66
1936	1298
968	946
154	2178
1386	1210
418	242
1650	1474
682	506
1914	1738
330	924
1562	2156
604	1188
1826	220
858	1452
2090	1100
1122	132
308	1364
1540	396
572	1628
1804	660
836	1892
2068	1078
484	110
1716	1342
748	374
1980	1606
1012	
44	
1276	

No. 180

1.	236210430
----	-----------

2.	236426874	506
3.	257644752	1794
4.	257861196	782
5.	1148672552	2070
6.	364818484	1058
7.	386465006	46
8.	1903972988	1334
9.	528299486	483
10.	547821886	1771
11.	472297784	759
12.	1148888996	2047
13.	408249458	1035
14.	322382728	667
15.	450884682	1955

No. 186

23	943
1311	2231
299	1219
1587	207
575	1495
1863	644
851	1932
2139	920
1288	2208
276	1196
1564	828
552	2116
1840	1104
184	92
1472	1380
460	368
1748	1656
736	805
2024	2093
1012	1081
161	69
1449	1357
437	989
1725	2277
713	1265
2001	253
345	1541
1623	529
621	1817
1909	966
897	2254
2185	1242
1173	230
322	1518
1610	1150
598	138
1886	1426
874	414
2162	1702
	690
	1978
	1127
	115
	1403

174 THE ART OF CALCULATION

391	336	1776	1775
1679	1680	720	675
	624	2064	2075
No. 187	1968	1176	975
1. 247453245	912	120	2375
2. 247679991	2256	1464	1275
3. 269907768	528	408	350
4. 270134514	1872	1752	1750
5. 1203345468	816		650
6. 382182606	2160	No. 194	2050
7. 404859429	1104	1. 258696060	950
8. 1994595642	48	2. 258933108	2350
9. 553444749	1392	3. 282170784	550
10. 573896349	504	4. 282407832	1950
11. 494777556	1848	5. 1258018384	850
12. 1203572214	792	6. 399546728	2250
13. 427680747	2136	7. 423253852	1150
14. 337727052	1080	8. 2085218296	50
15. 472345263	696	9. 578590012	1450
	2040	10. 599970812	525
	984	11. 517257328	1925
	2328	12. 1258255432	825
	1272	13. 447112036	2225
No. 193	216	14. 353071376	1125
24	1560	15. 493805844	725
1368	672		2125
312	2016		1025
1656	960		2425
600	2304		1325
1944	1248		225
888	864	No. 200	1625
2232	2208	25	700
1344	1152	1425	2100
288	96	325	1000
1632	1440	1725	2400
576	384	625	1300
1920	1728	2025	900
192	840	925	2300
1536	2184	2325	1200
480	1128	1400	100
1824	72	300	1500
768	1416	1700	400
2112	1032	600	1800
1056	2376	2000	875
168	1320	200	2275
1512	264	1600	1175
456	1608	500	75
1800	552	1900	1475
744	1896	800	1075
2088	1008	2200	2475
360	2352	1100	1375
1704	1296	175	275
648	240	1575	1675
1992	1584	475	575
936	1200	1875	1975
2280	144	775	1050
1224	1488	2175	2450
	432	375	1350

250	No. 219	2. 726	No. 240
1650		3. 1059	1. 755
1250	(Annex O to	4. 1392	2. 1310
150	Answers to	5. 1713	3. 1865
1550	No. 52)	6. 1896	4. 2420
450		7. 2229	5. 2975
1850	No. 222	8. 2562	6. 3280
750		9. 2883	7. 3805
2150	(Annex O to	10. 516	8. 4360
1225	Answers to	11. 699	9. 4915
125	No. 53)	12. 1032	10. 970
1525		13. 1353	11. 1275
425	No. 226	14. 1686	12. 1830
1825		15. 2019	13. 2355
	(Annex O to	16. 2202	14. 2910
	Answers to	17. 2523	15. 3465
	No. 56)	18. 2856	16. 3770
No. 201		19. 489	17. 4325
1. 269938875	No. 228	20. 822	18. 4880
2. 270186225			19. 905
3. 294433800	(Annex O to	No. 236	20. 1460
4. 294681150	Answers to	(Annex O to	No. 242
5. 1312691300	No. 60)	Answers to	(Annex O to
6. 416910850		No. 77)	Answers to
7. 441648275	No. 229		No. 106)
8. 2175840950		No. 237	
9. 603735275	1. 242	1. 564	No. 243
10. 626045275	2. 464	2. 1008	1. 846
11. 539737100	3. 686	3. 1452	2. 1512
12. 1312938650	4. 902	4. 1896	3. 2178
13. 466543325	5. 1124	5. 2340	4. 2844
14. 368415700	6. 1246	6. 2564	5. 3510
15. 515266425	7. 1462	7. 3008	6. 4176
	8. 1684	8. 3452	7. 4482
	9. 1906	9. 3892	8. 5106
No. 204	10. 322	10. 740	9. 5772
(Annex O to	11. 444	11. 964	10. 1038
Answers to	12. 666	12. 1408	11. 1704
No. 45)	13. 882	13. 1852	12. 2370
	14. 1104	14. 2296	13. 2676
No. 208	15. 1326	15. 2740	14. 3342
(Annex O to	16. 1442	16. 2964	15. 3966
Answers to	17. 1664	17. 3408	16. 4632
No. 46)	18. 1886	18. 3852	17. 5298
	19. 302	19. 696	18. 5964
No. 212	20. 524	20. 1140	19. 870
(Annex O to			20. 1536
Answers to	No. 232		
No. 47)	(Annex O to	No. 239	No. 244
	Answers to	(Annex O to	(Annex O to
	No. 61)	Answers to	Answers to
No. 215		No. 90)	No. 119)
(Annex O to	No. 233		
Answers to			
No. 50)	1. 393		

176 THE ART OF CALCULATION

No. 245

1. 917
2. 1694
3. 2471
4. 3248
5. 4025
6. 4802
7. 5579
8. 5866
9. 6587
10. 1064
11. 1841
12. 2618
13. 3395
14. 4172
15. 4459
16. 5236
17. 5957
18. 6734
19. 1211
20. 1988

No. 246

(Annex O to
Answers to
No. 131)

No. 247

1. 1128
2. 2016
3. 2904
4. 3792
5. 4680
6. 5568
7. 5976
8. 6864
9. 7752
10. 1368
11. 2256
12. 3144
13. 3552
14. 4440
15. 5328
16. 6216
17. 7104
18. 7992
19. 5928
20. 5216

No. 248

1. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

$$2. \frac{1}{18}, \frac{1}{18}, \frac{1}{18}$$

$$3. \frac{1}{6}, \frac{1}{6}, \frac{1}{6}$$

$$4. \frac{1}{12}, \frac{1}{12}, \frac{1}{12}$$

$$5. \frac{1}{24}, \frac{1}{24}, \frac{1}{24}$$

$$6. \frac{1}{36}, \frac{1}{36}, \frac{1}{36}$$

$$7. \frac{1}{48}, \frac{1}{48}, \frac{1}{48}$$

$$8. \frac{1}{60}, \frac{1}{60}, \frac{1}{60}$$

$$9. \frac{1}{72}, \frac{1}{72}, \frac{1}{72}$$

$$10. \frac{1}{84}, \frac{1}{84}, \frac{1}{84}$$

$$11. \frac{1}{96}, \frac{1}{96}, \frac{1}{96}$$

$$12. \frac{1}{108}, \frac{1}{108}, \frac{1}{108}$$

$$13. \frac{1}{120}, \frac{1}{120}, \frac{1}{120}$$

$$14. \frac{1}{132}, \frac{1}{132}, \frac{1}{132}$$

$$15. \frac{1}{144}, \frac{1}{144}, \frac{1}{144}$$

$$16. \frac{1}{156}, \frac{1}{156}, \frac{1}{156}$$

$$17. \frac{1}{168}, \frac{1}{168}, \frac{1}{168}$$

$$18. \frac{1}{180}, \frac{1}{180}, \frac{1}{180}$$

$$19. \frac{1}{192}, \frac{1}{192}, \frac{1}{192}$$

$$20. \frac{1}{204}, \frac{1}{204}, \frac{1}{204}$$

$$21. \frac{1}{216}, \frac{1}{216}, \frac{1}{216}$$

$$22. \frac{1}{228}, \frac{1}{228}, \frac{1}{228}$$

$$23. \frac{1}{240}, \frac{1}{240}, \frac{1}{240}$$

$$24. \frac{1}{252}, \frac{1}{252}, \frac{1}{252}$$

$$25. \frac{1}{264}, \frac{1}{264}, \frac{1}{264}$$

$$26. \frac{1}{276}, \frac{1}{276}, \frac{1}{276}$$

$$27. \frac{1}{288}, \frac{1}{288}, \frac{1}{288}$$

$$19. \frac{1}{18}, \frac{1}{18}, \frac{1}{18}$$

$$20. \frac{1}{20}, \frac{1}{20}, \frac{1}{20}$$

$$21. \frac{1}{22}, \frac{1}{22}, \frac{1}{22}$$

$$22. \frac{1}{24}, \frac{1}{24}, \frac{1}{24}$$

$$23. \frac{1}{26}, \frac{1}{26}, \frac{1}{26}$$

$$24. \frac{1}{28}, \frac{1}{28}, \frac{1}{28}$$

$$25. \frac{1}{30}, \frac{1}{30}, \frac{1}{30}$$

$$26. \frac{1}{32}, \frac{1}{32}, \frac{1}{32}$$

$$27. \frac{1}{34}, \frac{1}{34}, \frac{1}{34}$$

$$28. \frac{1}{36}, \frac{1}{36}, \frac{1}{36}$$

$$29. \frac{1}{38}, \frac{1}{38}, \frac{1}{38}$$

$$30. \frac{1}{40}, \frac{1}{40}, \frac{1}{40}$$

$$31. \frac{1}{42}, \frac{1}{42}, \frac{1}{42}$$

$$32. \frac{1}{44}, \frac{1}{44}, \frac{1}{44}$$

$$33. \frac{1}{46}, \frac{1}{46}, \frac{1}{46}$$

$$34. \frac{1}{48}, \frac{1}{48}, \frac{1}{48}$$

$$35. \frac{1}{50}, \frac{1}{50}, \frac{1}{50}$$

$$36. \frac{1}{52}, \frac{1}{52}, \frac{1}{52}$$

$$37. \frac{1}{54}, \frac{1}{54}, \frac{1}{54}$$

$$38. \frac{1}{56}, \frac{1}{56}, \frac{1}{56}$$

$$39. \frac{1}{58}, \frac{1}{58}, \frac{1}{58}$$

$$40. \frac{1}{60}, \frac{1}{60}, \frac{1}{60}$$

No. 251

1. 1368
2. 2367
3. 3366
4. 4365
5. 5364
6. 5823
7. 6822
8. 7821
9. 8757
10. 1656
11. 2655
12. 3114
13. 4113
14. 5112
15. 6111
16. 7056
17. 8055
18. 8514
19. 1413
20. 2412

No. 252

1. 121
2. 232
3. 343
4. 451
5. 562
6. 623
7. 731
8. 842

9. 953
10. 161
11. 222
12. 333
13. 441
14. 552
15. 663
16. 721
17. 832
18. 943
19. 151
20. 262

No. 253

1. $\frac{1}{18}$
2. $\frac{1}{18}$
3. $\frac{1}{18}$
4. $\frac{1}{18}$
5. $\frac{1}{18}$
6. $\frac{1}{18}$
7. $\frac{1}{18}$
8. $\frac{1}{18}$
9. $\frac{1}{18}$
10. $\frac{1}{18}$

No. 254

(Annex O to
Answers to
No. 148)

No. 255

1. 131
2. 242
3. 353
4. 464
5. 571
6. 632
7. 743
8. 854
9. 961
10. 172
11. 233
12. 344
13. 451
14. 562
15. 673
16. 734
17. 841
18. 952
19. 163
20. 274

No. 256

1. $1\frac{1}{2}$
2. $1\frac{1}{3}$
3. $1\frac{1}{6}$
4. $1\frac{1}{3}$
5. $1\frac{1}{6}$
6. $1\frac{1}{6}$
7. $1\frac{1}{6}$
8. $1\frac{1}{3}$
9. $1\frac{1}{3}$
10. $1\frac{1}{3}$

No. 257

(Annex O to
Answers to
No. 156)

No. 258

1. 141
2. 252
3. 363
4. 474
5. 585
6. 641
7. 752
8. 863
9. 974
10. 185
11. 241
12. 352
13. 463
14. 574
15. 685
16. 741
17. 852
18. 963
19. 174
20. 285

No. 259

1. $1\frac{1}{3}$
2. $1\frac{1}{3}$
3. $1\frac{1}{6}$
4. $1\frac{1}{3}$
5. $1\frac{1}{3}$
6. $1\frac{1}{3}$
7. $1\frac{1}{3}$
8. $1\frac{1}{3}$
9. $1\frac{1}{3}$
10. $1\frac{1}{3}$

No. 260

(Annex O to
Answers to
No. 165)

No. 261

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. $1\frac{1}{2}$
9. $1\frac{1}{2}$
10. $1\frac{1}{2}$

No. 262

1. 151
2. 262
3. 373
4. 484
5. 595
6. 656
7. 761
8. 872
9. 983
10. 194
11. 255
12. 366
13. 471
14. 582
15. 693
16. 754
17. 865
18. 976
19. 181
20. 292

No. 263

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. $\frac{1}{2}$
9. $\frac{1}{2}$
10. $\frac{1}{2}$
11. $\frac{1}{2}$

12. $\frac{1}{2}$
13. $\frac{1}{2}$
14. $\frac{1}{2}$
15. $\frac{1}{2}$
16. $\frac{1}{2}$
17. $\frac{1}{2}$
18. $\frac{1}{2}$
19. $\frac{1}{2}$
20. $\frac{1}{2}$
21. $\frac{1}{2}$
22. $\frac{1}{2}$
23. $\frac{1}{2}$
24. $\frac{1}{2}$
25. $\frac{1}{2}$
26. $\frac{1}{2}$
27. $\frac{1}{2}$
28. $\frac{1}{2}$
29. $\frac{1}{2}$
30. $\frac{1}{2}$

No. 264

(Annex O to
Answers to
No. 172)

No. 265

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. $\frac{1}{2}$
9. $\frac{1}{2}$
10. $\frac{1}{2}$

No. 266

1. 141
2. 252
3. 363
4. 474
5. 585
6. 696
7. 747
8. 851
9. 962
10. 173
11. 284
12. 395
13. 446
14. 557

15. 661
16. 772
17. 883
18. 994
19. 145
20. 256

No. 267

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $1\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $1\frac{1}{2}$
8. $1\frac{1}{2}$
9. $\frac{1}{2}$
10. $1\frac{1}{2}$

No. 268

(Annex O to
Answers to
No. 179)

No. 269

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. $\frac{1}{2}$
9. $\frac{1}{2}$
10. $\frac{1}{2}$

No. 270

1. 131
2. 242
3. 353
4. 464
5. 575
6. 686
7. 797
8. 838
9. 941
10. 152
11. 263
12. 374
13. 485

14. 596
15. 637
16. 748
17. 851
18. 962
19. 173
20. 284

No. 271

1. $\frac{3}{4}$
2. $1\frac{1}{4}$
3. $\frac{5}{8}$
4. $1\frac{1}{8}$
5. $\frac{1}{2}$
6. $1\frac{1}{8}$
7. $\frac{1}{2}$
8. $\frac{1}{4}$
9. $\frac{1}{2}$
10. $1\frac{1}{4}$

No. 272

(Annex O to
Answers to
No. 186)

No. 273

1. $\frac{2}{3}$
2. $\frac{1}{3}$
3. $\frac{1}{3}$
4. $\frac{1}{3}$
5. $\frac{1}{3}$
6. $\frac{1}{3}$
7. $\frac{1}{3}$
8. $\frac{1}{3}$
9. $\frac{2}{3}$
10. $\frac{1}{3}$

No. 274

1. 141
2. 252
3. 363
4. 474
5. 585
6. 696
7. 747
8. 858
9. 969
10. 171
11. 282

12. 393
13. 444
14. 555
15. 666
16. 777
17. 888
18. 999
19. 741
20. 652

No. 275

1. $\frac{1}{2}$
2. $1\frac{1}{4}$
3. $1\frac{1}{4}$
4. $1\frac{1}{4}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $1\frac{1}{4}$
8. $1\frac{1}{4}$
9. $\frac{1}{2}$
10. $\frac{1}{2}$

No. 276

(Annex O to
Answers to
No. 193)

No. 277

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. $\frac{1}{2}$
9. $\frac{1}{2}$
10. $\frac{1}{2}$

No. 278

1. 152
2. 263
3. 374
4. 485
5. 596
6. 647
7. 758

8. 869
9. 973
10. 184
11. 295
12. 346
13. 437
14. 568
15. 679
16. 784
17. 895
18. 946
19. 157
20. 268

No. 279

1. $\frac{5}{8}$
2. $1\frac{1}{8}$
3. $\frac{5}{8}$
4. $1\frac{1}{8}$
5. $1\frac{1}{8}$
6. $1\frac{1}{8}$
7. $\frac{5}{8}$
8. $\frac{5}{8}$
9. $\frac{5}{8}$
10. $1\frac{1}{8}$

No. 280

(Annex O to
Answers to
No. 200)

No. 281

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. $\frac{1}{2}$
9. $\frac{1}{2}$
10. $\frac{1}{2}$

No. 282

1. 2r86
2. 2r129
3. 2r108
4. 2r347
5. 2r456
6. 2r589

7. 2r312
8. 2r102
9. 2r208
10. 2r117
11. 3r13
12. 3r50
13. 3r105
14. 3r182
15. 3r285
16. 4r126
17. 4r200
18. 4r252
19. 4r282
20. 4r280

No. 283

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $1\frac{1}{2}$
5. $\frac{1}{2}$
6. $1\frac{1}{2}$
7. $1\frac{1}{2}$
8. $1\frac{1}{2}$
9. $\frac{1}{2}$
10. $1\frac{1}{2}$

No. 284

1. 1066
2. 1377
3. 1708
4. 2059
5. 2511
6. 2912
7. 1023
8. 1394
9. 1326
10. 1647
11. 1988
12. 2349
13. 2821
14. 992
15. 1353
16. 1734
17. 1586
18. 1917
19. 2268
20. 2639

No. 285

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$

4. $1\frac{1}{2}$
5. $1\frac{1}{2}$
6. $1\frac{1}{2}$
7. $1\frac{1}{2}$
8. $1\frac{1}{2}$
9. $1\frac{1}{2}$
10. $1\frac{1}{2}$

No. 286

1. 2r1
2. 2r29
3. 2r376
4. 2r551
5. 2r374
6. 3r378
7. 3r518
8. 3r680
9. 3r864
10. 3r17
11. 4r266
12. 4r225
13. 4r172
14. 4r93
15. 4r162
16. 5r90
17. 5r130
18. 5r148
19. 5r144
20. 5r119

No. 287

1. $1\frac{1}{2}$
2. $1\frac{1}{2}$
3. $1\frac{1}{2}$
4. $1\frac{1}{2}$
5. $1\frac{1}{2}$
6. $1\frac{1}{2}$
7. $1\frac{1}{2}$
8. $1\frac{1}{2}$
9. $1\frac{1}{2}$
10. $1\frac{1}{2}$

No. 288

1. 1470
2. 1872
3. 2294
4. 2736
5. 3198
6. 3772
7. 1344

8. 1806
9. 1820
10. 2232
11. 2664
12. 3116
13. 3588
14. 1312
15. 1764
16. 2236
17. 2108
18. 2520
19. 2952
20. 3404

No. 289

1. $1\frac{1}{2}$
2. $1\frac{1}{2}$
3. $1\frac{1}{2}$
4. $1\frac{1}{2}$
5. $1\frac{1}{2}$
6. $1\frac{1}{2}$
7. $1\frac{1}{2}$
8. $1\frac{1}{2}$
9. $1\frac{1}{2}$
10. $1\frac{1}{2}$

No. 290

1. 2r37
2. 2r771
3. 2r150
4. 2r85
5. 2r99
6. 3r46
7. 3r102
8. 3r170
9. 3r280
10. 3r402
11. 4r192
12. 4r235
13. 4r276
14. 4r285
15. 4r272
16. 5r67
17. 5r693
18. 5r564
19. 5r632
20. 5r97

No. 291

1. $1\frac{1}{2}$
2. $1\frac{1}{2}$

3. $1\frac{1}{2}$
4. $1\frac{1}{2}$
5. $1\frac{1}{2}$
6. $1\frac{1}{2}$
7. $1\frac{1}{2}$
8. $1\frac{1}{2}$
9. $1\frac{1}{2}$
10. $1\frac{1}{2}$

No. 292

1. 1892
2. 2385
3. 2898
4. 3431
5. 3984
6. 4557
7. 1683
8. 2236
9. 2332
10. 2835
11. 3358
12. 3901
13. 4464
14. 1617
15. 2193
16. 2756
17. 2772
18. 3510
19. 3818
20. 4371

No. 293

1. $1\frac{1}{2}$
2. $1\frac{1}{2}$
3. $1\frac{1}{2}$
4. $1\frac{1}{2}$
5. $1\frac{1}{2}$
6. $1\frac{1}{2}$
7. $1\frac{1}{2}$
8. $1\frac{1}{2}$
9. $1\frac{1}{2}$
10. $1\frac{1}{2}$

No. 294

1. 3r51
2. 3r69
3. 3r95
4. 3r32
5. 3r54
6. 4r226
7. 4r85
8. 4r864

9. 4r119
10. 4r208
11. 5r146
12. 5r288
13. 5r321
14. 5r465
15. 5r108
16. 6r125
17. 6r200
18. 6r77
19. 6r111
20. 6r310

No. 295

1. $1\frac{1}{2}$
2. $1\frac{1}{2}$
3. $1\frac{1}{2}$
4. $1\frac{1}{2}$
5. $1\frac{1}{2}$
6. $1\frac{1}{2}$
7. $1\frac{1}{2}$
8. $1\frac{1}{2}$
9. $1\frac{1}{2}$
10. $1\frac{1}{2}$

No. 296

1. 2332
2. 2916
3. 3520
4. 4144
5. 4788
6. 5452
7. 2006
8. 2684
9. 2862
10. 3456
11. 4070
12. 4704
14. 1972
15. 2596
16. 3599
17. 3392
18. 3996
19. 4620
20. 5264

No. 297

1. $1\frac{1}{2}$
2. $1\frac{1}{2}$
3. $1\frac{1}{2}$
4. $1\frac{1}{2}$

180 THE ART OF CALCULATION

5. $\frac{1}{12}$
6. $\frac{1}{12}$
7. $\frac{1}{12}$
8. $\frac{1}{12}$
9. $\frac{1}{12}$
10. $\frac{1}{12}$

No. 298

1. 5r219
2. 5r642
3. 5r312
4. 5r97
5. 5r106
6. 6r310
7. 6r150
8. 6r100
9. 6r609
10. 6r115
11. 7r65
12. 7r135
13. 7r235
14. 7r185
15. 7r64
16. 8r72
17. 8r125
18. 8r180
19. 8r380
20. 8r421

No. 299

1. $\frac{1}{12}$
2. $\frac{1}{12}$
3. $\frac{1}{12}$
4. $\frac{1}{12}$
5. $\frac{1}{12}$
6. $\frac{1}{12}$
7. $\frac{1}{12}$
8. $\frac{1}{12}$
9. $\frac{1}{12}$
10. $\frac{1}{12}$

No. 300

1. 2790
2. 3465
3. 4160
4. 4875
5. 5610
6. 6365
7. 2380
8. 3105

9. 3410
10. 4095
11. 4800
12. 5525
13. 6270
14. 2345
15. 3060
16. 3795
17. 4030
18. 4725
19. 5440
20. 6175

No. 301

1. $\frac{1}{12}$
2. $\frac{1}{12}$
3. $\frac{1}{12}$
4. $\frac{1}{12}$
5. $\frac{1}{12}$
6. $\frac{1}{12}$
7. $\frac{1}{12}$
8. $\frac{1}{12}$
9. $\frac{1}{12}$
10. $\frac{1}{12}$

No. 302

1. 6r10
2. 6r29
3. 6r38
4. 6r165
5. 6r651
6. 7r501
7. 7r307
8. 7r799
9. 7r646
10. 7r20
11. 8r189
12. 8r612
13. 8r325
14. 8r486
15. 8r17
16. 9r125
17. 9r135
18. 9r74
19. 9r85
20. 9r59

No. 303

1. $\frac{1}{12}$
2. $\frac{1}{12}$
3. $\frac{1}{12}$
4. $\frac{1}{12}$

5. $\frac{1}{12}$
6. $\frac{1}{12}$
7. $\frac{1}{12}$
8. $\frac{1}{12}$
9. $\frac{1}{12}$
10. $\frac{1}{12}$

No. 304

1. 3266
2. 4032
3. 4818
4. 5624
5. 6450
6. 7296
7. 2772
8. 3588
9. 3976
10. 4752
11. 5548
12. 6364
13. 7200
14. 2736
15. 3542
16. 4368
17. 4686
18. 5472
19. 6278
20. 7104

No. 305

1. $\frac{1}{12}$
2. $\frac{1}{12}$
3. $\frac{1}{12}$
4. $\frac{1}{12}$
5. $\frac{1}{12}$
6. $\frac{1}{12}$
7. $\frac{1}{12}$
8. $\frac{1}{12}$
9. $\frac{1}{12}$
10. $\frac{1}{12}$

No. 306

1. 6r706
2. 6r95
3. 6r37
4. 6r38
5. 6r40
6. 7r18
7. 7r118
8. 7r211
9. 7r346
10. 7r252
11. 8r28
12. 8r39

13. 8r404
14. 8r355
15. 8r626
16. 9r64
17. 9r301
18. 9r400
19. 9r500
20. 9r65

No. 307

1. $\frac{1}{12}$
2. $\frac{1}{12}$
3. $\frac{1}{12}$
4. $\frac{1}{12}$
5. $\frac{1}{12}$
6. $\frac{1}{12}$
7. $\frac{1}{12}$
8. $\frac{1}{12}$
9. $\frac{1}{12}$
10. $\frac{1}{12}$

No. 308

1. 3713
2. 4617
3. 5494
4. 6391
5. 7308
6. 8245
7. 3182
8. 4089
9. 4503
10. 5427
11. 6314
12. 7221
13. 8148
14. 3145
15. 4042
16. 4959
17. 5293
18. 6237
19. 7134
20. 8051

No. 309

1. $\frac{1}{12}$
2. $\frac{1}{12}$
3. $\frac{1}{12}$
4. $\frac{1}{12}$
5. $\frac{1}{12}$
6. $\frac{1}{12}$
7. $\frac{1}{12}$
8. $\frac{1}{12}$

9. $\frac{1}{2}$
10. $\frac{1}{2}$

No. 310

1. 7r129
2. 7r642
3. 7r711
4. 7r32
5. 7r232
6. 8r77
7. 8r444
8. 8r312
9. 8r147
10. 8r25
11. 9r27
12. 9r297
13. 9r358
14. 9r555
15. 9r609
16. 9r775
17. 9r862
18. 9r927
19. 9r150
20. 9r215

No. 311

1. $1\frac{2}{3}$
2. $1\frac{1}{2}$
3. $\frac{2}{3}$
4. $\frac{2}{3}$
5. $1\frac{1}{2}$
6. $1\frac{1}{2}$
7. $1\frac{1}{2}$
8. $1\frac{1}{2}$
9. $1\frac{1}{2}$
10. $1\frac{1}{2}$

No. 312

1. 4224
2. 5162
3. 6188
4. 7176
5. 8184
6. 9212
7. 3610
8. 4608
9. 5104
10. 6052
11. 7098
12. 8096
13. 9114
14. 3572

15. 4560
16. 5568
17. 5984
18. 6942
19. 8008
20. 9016

No. 313

1. $\frac{2}{3}$
2. $\frac{2}{3}$
3. $\frac{1}{3}$
4. $\frac{2}{3}$
5. $\frac{2}{3}$
6. $\frac{2}{3}$
7. $\frac{1}{3}$
8. $\frac{2}{3}$
9. $\frac{2}{3}$
10. $\frac{2}{3}$

No. 314

1. $\frac{2}{3}$
2. $1\frac{2}{3}$
3. $1\frac{2}{3}$
4. $1\frac{2}{3}$
5. $\frac{2}{3}$
6. $\frac{2}{3}$
7. $\frac{2}{3}$
8. $1\frac{2}{3}$
9. $\frac{2}{3}$
10. $\frac{2}{3}$

No. 315

1. 4655
2. 5664
3. 6693
4. 7742
5. 8811
6. 9405
7. 3744
8. 4753
9. 5782
10. 6831
11. 7505
12. 8544
13. 9603
14. 3822
15. 4851
16. 5605
17. 6624
18. 7663
19. 8722
20. 9801

No. 316

1. $\frac{1}{3}$
2. $\frac{1}{3}$
3. $\frac{1}{3}$
4. $\frac{1}{3}$
5. $\frac{1}{3}$
6. $\frac{1}{3}$
7. $\frac{1}{3}$
8. $\frac{1}{3}$
9. $\frac{1}{3}$
10. $\frac{1}{3}$

No. 317

1. $1\frac{1}{3}$
2. $1\frac{1}{3}$
3. $\frac{1}{3}$
4. $1\frac{1}{3}$
5. $1\frac{1}{3}$
6. $1\frac{1}{3}$
7. $\frac{1}{3}$
8. $\frac{1}{3}$
9. $1\frac{1}{3}$
10. $1\frac{1}{3}$

No. 318

1. $\frac{1}{3}$
2. $\frac{1}{3}$
3. $\frac{1}{3}$
4. $\frac{1}{3}$
5. $\frac{1}{3}$
6. $\frac{1}{3}$
7. $\frac{1}{3}$
8. $\frac{1}{3}$
9. $\frac{1}{3}$
10. $\frac{1}{3}$

No. 319

1. 41
2. 51
3. 61
4. 71
5. 81
6. 91
7. 31
8. 41
9. 51
10. 61
11. 71
12. 81
13. 91

14. 31
15. 41
16. 51
17. 61
18. 71
19. 81
20. 91

No. 320

1. $\frac{1}{3}$
2. $\frac{1}{3}$
3. $\frac{1}{3}$
4. $\frac{1}{3}$
5. $\frac{1}{3}$
6. $\frac{1}{3}$
7. $\frac{1}{3}$
8. $\frac{1}{3}$
9. $\frac{1}{3}$
10. $\frac{1}{3}$

No. 321

1. $\frac{1}{3}$
2. $\frac{1}{3}$
3. $\frac{1}{3}$
4. $\frac{1}{3}$
5. $\frac{1}{3}$
6. $\frac{1}{3}$
7. $\frac{1}{3}$
8. $\frac{1}{3}$
9. $\frac{1}{3}$
10. $\frac{1}{3}$

No. 322

1. 42
2. 52
3. 62
4. 72
5. 82
6. 92
7. 32
8. 42
9. 52
10. 62
11. 72
12. 82
13. 92
14. 32
15. 42
16. 52
17. 62
18. 72
19. 82

20. 92

No. 323

1. $1\frac{1}{4}$
2. $1\frac{1}{4}$
3. $1\frac{1}{4}$
4. $1\frac{1}{4}$
5. $1\frac{1}{4}$
6. $1\frac{1}{4}$

No. 324

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. $\frac{1}{2}$
9. $\frac{1}{2}$
10. $\frac{1}{2}$

No. 325

1. 43
2. 53
3. 63
4. 73
5. 83
6. 93
7. 33
8. 43
9. 53
10. 63
11. 73
12. 83
13. 93
14. 33
15. 43
16. 53
17. 63
18. 73
19. 83
20. 93

No. 327

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$

6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. $\frac{1}{2}$
9. $\frac{1}{2}$
10. $\frac{1}{2}$

No. 328

1. 44
2. 54
3. 64
4. 74
5. 84
6. 94
7. 34
8. 44
9. 54
10. 64
11. 74
12. 84
13. 94
14. 34
15. 44
16. 54
17. 64
18. 74
19. 84
20. 94

No. 330

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. $\frac{1}{2}$
9. $\frac{1}{2}$
10. $\frac{1}{2}$

No. 331

1. 45
2. 55
3. 65
4. 75
5. 85
6. 95
7. 35
8. 45
9. 55
10. 65
11. 75
12. 85

13. 95
14. 35
15. 45
16. 55
17. 65
18. 75
19. 85
20. 95

No. 332

1. 46
2. 56
3. 66
4. 76
5. 86
6. 96
7. 36
8. 46
9. 56
10. 66
11. 76
12. 86
13. 96
14. 36
15. 46
16. 56
17. 66
18. 76
19. 86
20. 96

No. 333

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. $\frac{1}{2}$
9. $\frac{1}{2}$
10. $\frac{1}{2}$

No. 334

1. 47
2. 57
3. 67
4. 77
5. 87
6. 97
7. 37
8. 47
9. 57

10. 67
11. 77
12. 87
13. 97
14. 37
15. 47
16. 57
17. 67
18. 77
19. 87
20. 97

No. 335

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{2}$
4. $\frac{1}{2}$
5. $\frac{1}{2}$
6. $\frac{1}{2}$

No. 336

1. 48
2. 58
3. 68
4. 78
5. 88
6. 98
7. 38
8. 48
9. 58
10. 68
11. 78
12. 88
13. 98
14. 38
15. 48
16. 58
17. 68
18. 78
19. 88
20. 98

No. 337

1. 49
2. 59
3. 69
4. 79
5. 89
6. 99
7. 39
8. 49
9. 59
10. 69

11. 79
12. 89
13. 99
14. 39
15. 49
16. 59
17. 69
18. 79
19. 89
20. 99

No. 338

1. $12\frac{1}{2}$
2. $37\frac{1}{2}$
3. $62\frac{1}{2}$
4. $87\frac{1}{2}$
5. $33\frac{1}{2}$
6. $66\frac{1}{2}$
7. $16\frac{1}{2}$
8. $83\frac{1}{2}$
9. .20
10. .40
11. .60
12. .80

No. 339

1. 2886
2. 5994
3. 9268
4. 12818
5. 17081
6. 19584
7. 23793
8. 28288
9. 24466
10. 4104

No. 340

1. $.06\frac{1}{2}$
2. $.18\frac{1}{2}$
3. $.31\frac{1}{2}$
4. $.43\frac{1}{2}$
5. $.56\frac{1}{2}$
6. $.68\frac{1}{2}$
7. $.81\frac{1}{2}$
8. $.93\frac{1}{2}$
9. $.08\frac{1}{2}$
10. $.41\frac{1}{2}$
11. $.58\frac{1}{2}$
12. $.91\frac{1}{2}$
13. $.03\frac{1}{2}$
14. $.04\frac{1}{2}$

No. 341

1. 4235
2. 8352
3. 12691
4. 17138
5. 21918
6. 25543
7. 30702
8. 36206
9. 33355
10. 5796

No. 342

1. \$17887
2. \$9818
3. 9865
4. 25775
5. 39540
6. 23332
7. 17313
8. 31383
9. \$14822.40
10. 243062

No. 343

1. 5764
2. 10890
3. 16238
4. 21808
5. 27408
6. 30968
7. 37893
8. 44408
9. 42284
10. 7740

No. 344

1. .0625
2. .1875
3. .3125
4. .4375
5. .5625
6. .6875
7. .8125
8. .9375
9. $.0833\frac{1}{3}$
10. $.4166\frac{2}{3}$
11. $.5833\frac{1}{3}$
12. $.9166\frac{2}{3}$
13. $.0312\frac{1}{2}$
14. $.0416\frac{1}{2}$

No. 345

1. 7473
2. 13608
3. 19965
4. 26544
5. 33345
6. 37178
7. 44368
8. 52643
9. 51622
10. 9990

No. 346

1. \$99.84
2. 96256
3. \$117.76
4. 98304
5. 1728
6. \$675.84
7. \$8120.60
8. \$30402.55

No. 347

1. 9362
2. 16506
3. 23872
4. 31460
5. 39270
6. 43952
7. 51748
8. 60168
9. 60946
10. 12222

No. 348

1. .03125
2. .09375
3. .15625
4. .21875
5. .28125
6. .34375
7. .40625
8. .46875
9. .53125
10. .59375
11. .65625
12. .71875
13. .78125
14. .84375
15. .90625
16. .96875
17. .04167

18. .20833
19. .29167
20. .45833
21. .54167
22. .70833
23. .79167
24. .95833

No. 349

1. 10011
2. 18144
3. 26499
4. 35076
5. 43875
6. 52896
7. 57519
8. 66378
9. 68302
10. 12456

No. 350

1. \$424575
2. \$84770
3. \$733779.50
4. \$26863.20
5. \$830062.74
6. \$526.32
7. \$981088
8. \$9603
9. \$1007010

No. 351

1. 10349
2. 19602
3. 28946
4. 38512
5. 48300
6. 58310
7. 68542
8. 72906
9. 74339
10. 12312

No. 353

1. 12408
2. 22428
3. 33033
4. 43608
5. 54405
6. 65424

184 THE ART OF CALCULATION

7. 70965
8. 82368
9. 85272
10. 15219

No. 354

1. \$525
2. \$756
3. \$384
4. \$810
5. \$5400
6. \$900
7. \$13000
8. \$14700
9. \$7200
10. \$1600
11. \$630
12. \$12800
13. \$1200
14. \$1200
15. \$1200

No. 355

1. 14440
2. 25248
3. 36278
4. 47530
5. 59004
6. 61465
7. 72768
8. 84293
9. 95354
10. 19206

No. 357

1. 11211
2. 24642
3. 40051
4. 57902
5. 77691
6. 92412
7. 116081
8. 142272
9. 170321
10. 29032

No. 358

1. \$247715.70
2. \$243540
3. \$60226335
4. \$1087638.75

5. \$5209451.52
6. \$131602.24
7. \$40102686.72
8. \$8710669

No. 359

1. 24442
2. 49184
3. 76146
4. 104632
5. 136004
6. 156996
7. 191522
8. 229024
9. 268746
10. 47012

No. 361

1. 39693
2. 75746
3. 114019
4. 154512
5. 195853
6. 223096
7. 269709
8. 318542
9. 368063
10. 67596

No. 362

1. 138138
2. 115596
3. 74556
4. 186960
5. 89301
6. 235872
7. 119782
8. 73248
9. 193256

No. 363

1. 56964
2. 104328
3. 153912
4. 205716
5. 259740
6. 291014
7. 348928
8. 409062
9. 471416
10. 91390

No. 364

1. 210
2. 342
3. 255
4. 240
5. 195
6. 247
7. 272
8. 224
9. 361

No. 365

1. 76255
2. 134930
3. 195825
4. 258940
5. 324275
6. 364080
7. 429965
8. 501400
9. 575055
10. 115430

No. 366

1. \$56496
2. \$799018
3. \$5663152
4. \$410091.55
5. \$453952.95
6. \$36033.25
7. \$530895.75
8. \$1043606.30

No. 367

1. 85446
2. 155232
3. 227238
4. 301464
5. 377910
6. 456576
7. 497502
8. 575276
9. 659932
10. 120408

No. 368

1. \$139510.50

2. \$147804.75
3. \$158233.30
4. \$131011.65
5. \$452339.40
6. \$754503.75
7. \$151524.65
8. \$238939.80

No. 369

1. 92617
2. 173514
3. 256631
4. 341968
5. 429525
6. 519302
7. 611299
8. 651126
9. 740567
10. 121144

No. 370

1. 5476
2. 8649
3. 6724
4. 4096
5. 1444
6. 12544
7. 15376
8. 21316
9. 28224
10. 38809
11. 1236544
12. 1471369
13. 1726596
14. 2298256
15. 2954961

No. 371

1. 113928
2. 206136
3. 300564
4. 397212
5. 496080
6. 597168
7. 648396
8. 753324
9. 860472
10. 153558

No. 372

1. 7616

2. 12561
3. 15824
4. 22425
5. 40716
6. 42749
7. 421056
8. 224196
9. 198989

No. 373

1. 138168
2. 241697
3. 347446
4. 455415
5. 565604
6. 620473
7. 734502
8. 850751
9. 962297
10. 183816

No. 374

1. 8556
2. 4030
3. 7308
4. 8924
5. 45795
6. 100152
7. 173888
8. 264171
9. 837221

No. 375

1. 2025
2. 3025

3. 4225
4. 5625
5. 7225
6. 9025
7. 13225
8. 18225
9. 24025
10. 30625
11. 38025
12. 99225
13. 112225
14. 126025
15. 140625

No. 376

1. 621
2. 2009
3. 1224
4. 11021
5. 13216
6. 24024
7. 30616
8. 27209
9. 38016

No. 377

1. 275625
2. 390625
3. 680625
4. 1050625
5. 1500625
6. 1755625
7. 2640625
8. 2975625

9. 3330625
10. 3705625

No. 378

1. 4896
2. 6391
3. 8084
4. 12019
5. 16851
6. 22484
7. 25536
8. 32351
9. 36036

No. 379

1. 90 $\frac{1}{2}$
2. 112 $\frac{1}{2}$
3. 160 $\frac{1}{2}$
4. 339 $\frac{1}{2}$
5. 12 $\frac{1}{2}$
6. 3681 $\frac{1}{2}$
7. 1625 $\frac{1}{2}$
8. 650 $\frac{1}{2}$
9. 28 $\frac{1}{2}$
10. 72 $\frac{1}{2}$
11. 42 $\frac{1}{2}$
12. 152 $\frac{1}{2}$

No. 380

1. 276
2. 800
3. 929 $\frac{1}{2}$
4. 950

5. 2552
6. 5952
7. 1422
8. 2100
9. 3363

No. 381

1. 23.2
2. 45
3. 36
4. 3.5
5. 5.12
6. 13.05
7. 10.18
8. 61.2
9. 77.6

No. 382

1. 2744
2. 19683
3. 35937
4. 97336
5. 205379
6. 238328
7. 274625
8. 357911
9. 389017
10. 592704
11. 636056
12. 681472
13. 857375
14. 912673
15. 970299

